ABSTRACT. The main result of this paper represents a generalization of Walsh's theorem to arbitrary topological linear spaces. This allows us to show, in particular, that the Stone-Weierstrass theorem is actually one concerning simultaneous interpolation and approximation and not just one of approximation alone. In other corollaries, we will exhibit some of the many applications of these results, including a few which establish the existence of "rules" (e.g. quadrature, differentiation, etc.) which are uniformly good over a linear space of functions and which are at the same time exact over a certain subspace.