ABSTRACT. Let $\{G_k\}$ be a sequence of linear subspaces of a normed linear space E. We want to characterize those sequences $\{g_k\}$ in E, with $g_k \in G_k$ $(k = 1, 2, ...)$, for which there exists an $x \in E$ such that

$$g_k \in P_{G_k}(x) \qquad (k = 1, 2, ...),$$

where $P_G(x)$ denotes the set of all elements of best approximation to x from G, i.e., the set

$$\{g_0 \in G \mid ||x - g_0|| = \inf_{g \in G} ||x - g||\}.$$

This general study was motivated by a problem posed by T.J. Rivlin [5] who asked: Characterize (intrinsically) those n-tuples of polynomials $(p_0, p_1, ..., p_{n-1})$, with degree of p_j equal to j for all j, for which there exists an $x \in C[0, 1]$ such that the polynomial of best approximation to x, from the space of polynomials P_j of degree at most j, $\Pi_{P_j}(x)$, is p_j $(j = 0, 1, ..., n - 1)$. What is the answer in the particular case where $n = 2$?

We can give a complete (but not intrinsic) characterization of our general problem (Theorem 1). This allows us, for example, to get a complete characterization of the general problem in a Hilbert space when the subspaces G_k are increasing (Theorem 5), or when they are decreasing (Theorem 6). In $C[0, 1]$, we answer Rivlin's second question as follows. Given $p_j \in P_j (j = 0, 1)$, there exists $x \in C[0, 1]$ such that $\Pi_{P_j}(x) = p_j$ $(j = 0, 1)$ if and only if $p_1 - p_0$ is either identically zero or changes sign once in $[0, 1]$.