ABSTRACT. The concept of a sun, introduced by Efimov and Steckin in [10], has proved to be rather important in the general theory of approximation in normed linear spaces. We mention only the following results:

(1) Every convex set is a sun, and in smooth spaces every proximinal sun is convex.

(2) In finite-dimensional normed linear spaces, every Chebyshev set is a sun (cf. Vlasov [14]).

In an earlier paper, the authors [1] generalized this concept to that of a “moon”. They showed that in certain spaces — the so-called MS-spaces — moons and suns coincide. They also showed that each of the following spaces is an MS-space. (1) $C_0(T), T$ locally compact; (2) $L_1(S, \Sigma, u)$, where $(S, \Sigma, u)$ is a $\sigma$-finite measure space; (3) Every finite-dimensional space whose unit ball is a polyhedron.

The main result of the paper is the following theorem. Let $V$ be a subset of the real normed linear space $X$. Then each of the following conditions implies the subsequent one: (1) $V$ is a sun; (2) The metric projection onto $V, P_V$, is “outer radially lower semicontinuous”; (3) For each $x \in X$, every local minimum of the functional $v \mapsto ||x - v||$ on $V$ is a global minimum; (4) $V$ is a moon. Moreover, in a MS-space, all these conditions are equivalent.