ABSTRACT. Recently, Dunham [5] has given what appears to be the first example of a Chebychev set in a normed linear space which is not a sun. (Actually, such an example is implicit in an earlier paper of Dunham [6, p. 383].) The initial motivation behind this study was the hope of constructing a Chebychev set which is not a sun in a Hilbert space. Such a set would answer the nearly forty-year-old problem in approximation theory of whether each Chebychev set in a Hilbert space must be convex (or equivalently a sun).

There are several equivalent conditions in order that an EU-regular Chebychev set be a sun (Theorem 1.17 and Remark 1.18). They can be stated in terms of certain important concepts in approximation theory such as approximatively compact, boundedly compact, boundedly connected, and continuous metric projection. The most useful of these equivalences for our purposes is (Theorem 1.17) that the mapping from the parameter space to the elements of the EU-regular set be continuous. As an application of this result, we show (Theorem 2.2) that *in the space $C_0(T)$, where T is locally compact Hausdorff, there exists an EU-regular Chebychev set which is not a sun if and only if T is $\sigma-$compact and T contains a nonempty nowhere dense compact G_δ.* Consequently, (Corollary 2.6) *if T is an infinite compact metric space, then $C(T)$ contains a Chebychev set which is not a sun.* (The proof is constructive; such a set may be explicitly exhibited.)