ABSTRACT. In this note we give an elementary proof of the Stone-Weierstrass theorem. The proof depends only on the definitions of compactness (“each open cover has a finite subcover”) and continuity (“the inverse images of open sets are open”), two simple consequences of these definitions (i.e. “a closed subset of a compact space is compact,” and “a positive continuous function on a compact set has a positive infimum”), and the elementary Bernoulli inequality:

\[(1 + h)^n \geq 1 + nh \quad (n = 1, 2, \ldots)\]

if \(h \geq -1\).