ABSTRACT. We study a special dual form of a convex minimization problem in a Hilbert space, which is formally suggested by Fenchel duality and is useful for Dykstra's algorithm. For this special duality problem, we prove that the strong duality holds if and only if the collection of underlying constraint sets $\{C_1, ..., C_m\}$ has the "strong conical hull intersection property" (or strong CHIP). That is, for each $x \in \cap_1^m C_i$, we have $(\cap_1^m C_i - x)^0 = \sum_1^m (C_i - x)^0$, where D^0 denotes the dual cone of D. In general, we can establish weak duality for a convex minimization problem in a Hilbert space by perturbing the constraint sets so that the perturbed sets have strong CHIP. This generalizes a result of Gaffke and Mathar [6].