ABSTRACT. Let X be a (real) Hilbert space, C be a closed convex subset, and $H_i = \{x \in X | \langle x, h_i \rangle \leq b_i\}$ $(i = 1, 2, ..., m)$ be a finite collection of half-spaces. Under the assumption $K := C \cap (\cap_1^m H_i)$ is not empty, the problem of characterizing the best approximation from K to any $x \in X$ is considered. The "strong conical hull intersection property" (strong CHIP), which was introduced by us in 1997, is shown to be both necessary and sufficient for the following "perturbation property" to hold: For each $x \in X$, an element $x_0 \in K$ satisfies $x_0 = P_K(x)$ if and only if $x_0 = P_C(x - \sum_1^m \lambda_i h_i)$ for some scalars $\lambda_i \geq 0$ with $\lambda_i[\langle x_0, h_i \rangle - b_i] = 0$ for each i. Here $P_D(z)$ denotes the unique best approximation from D to z. In other words, determining the best approximation from the set K to any point is equivalent to the (generally easier) problem of determining the best approximation from the set C to a perturbation of that point. Moreover, even when the strong CHIP does not hold, the perturbation property still holds, except now C must be replaced by a certain convex extremal subset of C. We also show that the strong CHIP is weaker than any of the weak Slater conditions that one can naturally impose on the sets in question. These results generalize the main results of our 1997 paper [*J. Approx. Theory*, 90, pp. 385–444] and hence those of several other papers as well.