ABSTRACT. There is an intimate relationship between (1) the set of all Tietze extensions of a given continuous function on a compact subset S of a locally compact Hausdorff space T to all of T, and (2) the set of all best approximations to elements of $C_0(T)$ from the ideal M in $C_0(T)$ consisting of those functions which vanish on S. This relation is used, for example, to deduce that the Tietze extension map has a linear selection if and only if the metric projection onto M has a linear selection. It is known that the former holds whenever T is metrizable.