ABSTRACT. There is an intimate relationship between (1) the set of all Tietze
extensions of a given continuous function on a compact subset $S$ of a locally compact
Hausdorff space $T$ to all of $T$, and (2) the set of all best approximations to elements
of $C_0(T)$ from the ideal $M$ in $C_0(T)$ consisting of those functions which vanish on $S$.
This relation is used, for example, to deduce that the Tietze extension map has a linear
selection if and only if the metric projection onto $M$ has a linear selection. It is known
that the former holds whenever $T$ is metrizable.