Ultrasonic Wave Propagation in Multilayer Ceramic Capacitor Structures Containing Cracks

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ABSTRACT

A theoretical model has been developed to analyze attenuation and velocity variation of ultrasonic waves propagating in a multilayer ceramic capacitor containing multiple cracks. Effects of alternating ceramic and electrode structures are calculated using the T-matrix method, while the influence of cracks is analyzed based on scattering theory. In the case of waves going through multiple split cracks, wave velocity decreases while attenuation increases with crack density. This model may be used as an indicator to construct a nondestructive testing (NDT) technique. Finite element simulations on various prototype structures with different crack densities and distributions further validated the theoretical results.

Keywords: *wave propagation, ultrasonic attenuation, multilayer structures, crack, multilayer ceramic capacitor.*

INTRODUCTION

Split cracks are common in multilayer ceramic capacitors. Due to thermal expansion mismatch between metal and ceramic components, sintering causes mechanical stresses that trigger cracks at interfaces (Freer et al., 1993). Current nondestructive testing (NDT) techniques for detecting such cracks include X-ray radiography (Bray and McBride, 1992) and piezoelectricity testing (Ousten et al., 1998). Each technique has certain limitations. Most importantly, resolution becomes a real challenge as crack size becomes less than 10 µm. Ultrasonic testing has been one of the most powerful methods for decades, but there is a conflict between resolution and penetration depth in ultrasonic techniques. Low frequency waves have low resolution while high frequency waves have shallow penetration depth. In addition, multilayer structures also present extra difficulties for ultrasonic imaging when cracks are small. The most advanced ultrasonic technique today can only image discontinuities larger than 20 µm. In order to resolve this conflict and utilize more ultrasonic information, we analyze the attenuation and velocity variations of ultrasonic waves going through multilayer ceramic capacitors containing different cracks.

The objective of this work is to show that the velocity and attenuation of ultrasonic waves are directly correlated to the crack density. There have been some preliminary analyses on this topic (Borovikov and Fradkin, 2003; Kitahara and Takahashi, 2003; Yang and Turner, 2003). However, they only investigated simple, single-crack situations. In our previous paper, ultrasonic wave propagation in multilayer structures containing delaminations was studied, and the relationship between ultrasonic wave characteristics and the number of layers as well as the influence of delamination density was derived (Zhu and Cao, 2006). An extension of same strategy is used to treat multilayer ceramic capacitors having 10 to 100 μm magnitude small cracks. In the process of capacitor fabrication, finding such small cracks is critical for evaluation of reliability. The treatment contains two parts: first, the T-matrix technique is employed to deal with the periodic nature of multilayer structures; then, several established approaches are applied to study the scattering problem. In order to verify the validity of the theoretical results, simulations using commercial software are performed. All calculations and simulations are based on longitudinal waves. For simplicity, cracks are assumed to be randomly distributed, but having similar sizes and shapes. Their unit normals are assumed to be coplanar. The crack densities are assumed to be small to reflect practical situations.

THEORETICAL MODEL

The theoretical treatment of wave scattering by discontinuities is based on the model given in Zhang and Gross (1993) and the mathematical approaches given in Zhang and Achenbach (1988) and Saha and Roy (2002). The scattering cross section of a single crack is numerically calculated using the boundary element method. The attenuation coefficient is then determined from energy calculations. When an ultrasonic wave propagating in an isotropic solid material encounters cracks, part of the incident wave is scattered or reflected, while a small portion will transmit through. The traction-free boundary condition at the surface is given by:

$$\sigma_{\mathrm{in}}(x)+\sigma_{\mathrm{tm}}(x)+\sigma_{\mathrm{sc}}(x)=0 \tag{1}$$

where

σ_{in} = the stress component of the incidence wave

σ_{tm} = the stress component of the transmitted wave

σ_{sc} = the stress component of the wave scattered from the crack.

According to Zhang and Gross (1993), the displacement component of a scattered wave is:

$$u_{\mathrm{sc}}=\frac{e^{i\left(k+\frac{\pi}{4}\right)}}{\sqrt{8\pi kr}}\,g(\theta) \tag{2}$$

where

k = the wave number of the incident wave

r = half of the crack length

θ = the angle between the incident wave and crack orientation.

The amplitude of scattered wave g is dependent on θ. From Equation 1 and the calculations in Zhang (1988), g can be derived for a single crack from the following equation:

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(3)
$$g(\theta)=-ik\left[2\kappa^{-2}\hat{x}_{\alpha}r\cos\theta+\left(1-2\kappa^{-2}\right)\delta_{\alpha 2}\right]$$
$$\frac{\left(Z_{\mathrm{m}}-Z_{\mathrm{air}}\right)}{\left(Z_{\mathrm{m}}+Z_{\mathrm{air}}\right)}\int_{-\pi}^{+\pi}e^{-ik_{L}\hat{x}_{\gamma}}\Delta u_{\alpha}(r\sin\theta)\,dx_{1}$$

where

$\delta_{\alpha\beta}$ is the Kronecker symbol

Δu_{α} is the crack opening displacement, and can be solved by using the boundary element method (Zhang and Achenbach, 1988) and the perturbation technique (Saha and Roy, 2002)

$\hat{x}$ is a unit vector in the direction of the incident wave

Z_{m} and Z_{air} are the acoustic impedances of capacitor material and air.

For an object containing randomly distributed cracks, the average forward scattering wave amplitude may be obtained by taking the average of $g(\theta)$ over all possible wave incidence angles.

(4)
$$g=\frac{1}{2\pi}\int_{0}^{2\pi}g(\theta)\,d\theta$$

When transmitting in a multilayer structure with cracks, the ultrasonic wave is influenced by both the layer boundaries and the scattering from cracks. The T-matrix technique is brought in to treat the multilayer effects (Zhu and Cao, 2006). For a multilayer capacitor containing N ceramic layers and $N+1$ electrode layers (Figure 1), the attenuation coefficient is given by:

(5)
$$[T1(N)]=[T_{b}][T_{a+b}]\cdots[T_{N(a+b)-a}][T_{N(a+b)}]$$

where

T denotes the transmission matrix between two adjacent layers

a and b describe the thickness of the ceramic and electrode layers.

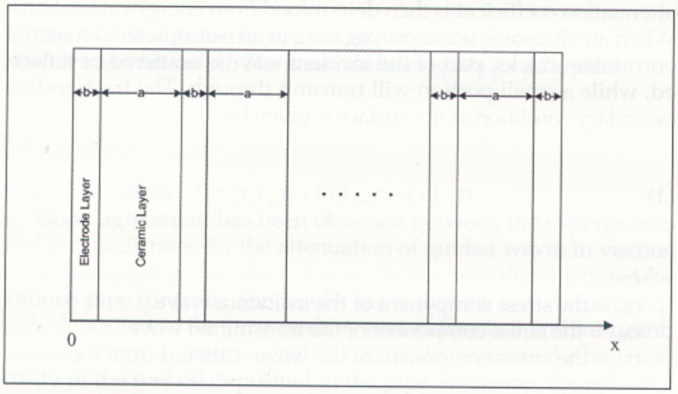

Figure 1 — Cross-sectional view of a periodic electrode/ceramic structure.

The effect of wave scattering from multiple cracks can be derived from Foldy (1945). After going through multiple cracks, the output ultrasound has effective wave number k':

(6)
$$k'r=\sqrt{(kr)^{2}+Ef}=kr+\frac{E}{2}\frac{\mathrm{Re}(g)}{k}+i\frac{E}{2}\frac{\mathrm{Im}(g)}{k}$$

where

k is the corresponding wave number in a perfect medium

E is the crack density parameter given by Budiansky and O'Connell (1976)

(7)
$$E=\frac{4}{\pi}nr^{2}$$

where

n indicates the total number of cracks in the structure.

By solving Equation 6, the relative phase velocity and attenuation coefficient are presented:

(8)
$$\frac{V}{V'}=\mathrm{Re}\left(\frac{k'}{k}\right)$$

(9)
$$\alpha'=\frac{2*\mathrm{Im}(k'r)}{r}$$

where

V and V' are the wave velocities in a perfect and anomalous medium

α' is the wave attenuation coefficient due to the scattering from cracks.

Using the area integral (Zhu and Cao, 2006) to combine the solution obtained from the scattering process with the results of Equation 5 above, the total attenuation coefficient and velocity relationship between incident and transmitted waves can be calculated.

NUMERICAL RESULTS AND DISCUSSIONS

Numerical calculations based on the theoretical derivations have been conducted for several scenarios. The multilayer ceramic capacitor structure is assumed to be composed of ten Ag layers and nine $BaTiO_3$ layers. Electrode and ceramic layer thicknesses are 2 and 8 µm, respectively. Capacitor volume is normalized to one. Poisson's ratios of Ag and $BaTiO_3$ are respectively set at 0.33 and 0.24. Acoustic impedances of involved components are $Z_{\mathrm{ceramic}}=2.4\times10^{7}$ kg · s^{-1} · m^{-2}, $Z_{\mathrm{electrode}}=3.8\times10^{7}$ kg · s^{-1} · m^{-2} and $Z_{\mathrm{air}}=0.4\times10^{3}$ kg · s^{-1} · m^{-2}. All cracks are assumed to be 25 µm long and have similar shape. Finite element analysis has been performed using commercial software to verify the theoretical results. Simulation is designed to use a three-dimensional enhanced strain method in the frequency domain for faster execution. Boundary admittance coefficients are specified at boundary element surfaces, while no surface activation flag is applied. Pressure is applied at the left surface of the capacitor (Figure 1), while the pressure and particle velocity data are collected at the right surface. Attenuation can be obtained by comparing the pressure difference at two sides. The simulated multilayer ceramic capacitor structures include different multiple discontinuity configurations and are subjected to different loading conditions.

Theoretical results and simulation outputs are shown in Figures 2 through 5. Figure 2 presents the relationship between crack density and the ratio of incident and output wave phase velocity at different frequencies. No wave velocity change takes place if no discontinuity exists. It is clearly shown that the effective wave velocity will decrease when discontinuity density increases. The same situation applies when there is an increase in crack size. However, phase velocity doesn't decrease linearly with frequency. As shown in Figure 3, steep frequency dependency occurs between 30 and 200 MHz.

As anticipated, the attenuation coefficient increases with crack density. Results of 18 MHz and 54 MHz incident waves are presented in Figure 4. In the low frequency regime, attenuation increases slowly with the increase of crack density. The signature of the attenuation is quite different at 54 MHz. It is very interesting to see a fall of the attenuation coefficient at 54 MHz. This decrease is due to the alternating layer structure of multilayer ceramic capacitor, which produces quasi-passbands and quasi-stopbands clearly illustrated in Figure 5, so that the effective attenuation oscillates with frequency. Waves can pass through at certain frequencies, without other attenuation sources, while they are strongly attenuated at other frequencies. This phenomenon adds to the frequency dependence of the attenuation caused by crack scattering. Figure 4 also shows that the

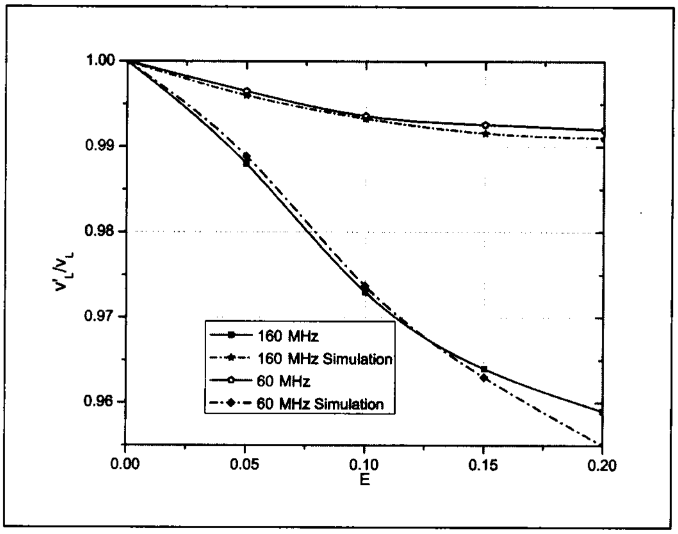

Figure 2 — Normalized wave velocity versus discontinuity density at different frequencies.

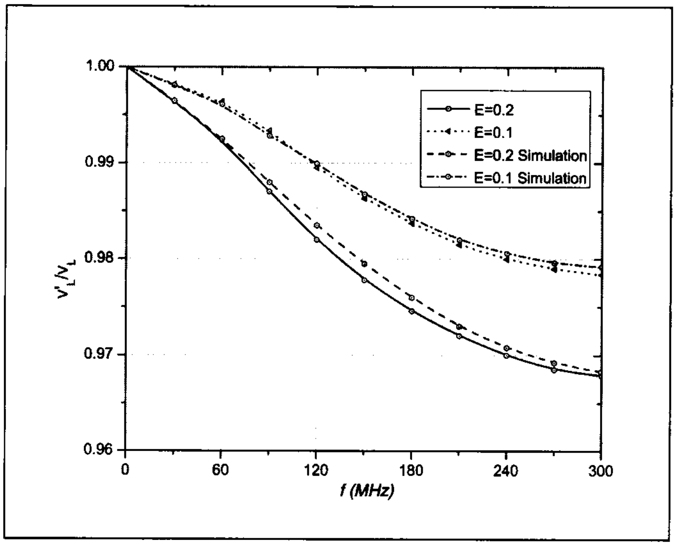

Figure 3 — Normalized wave velocity versus frequency for different discontinuity densities.

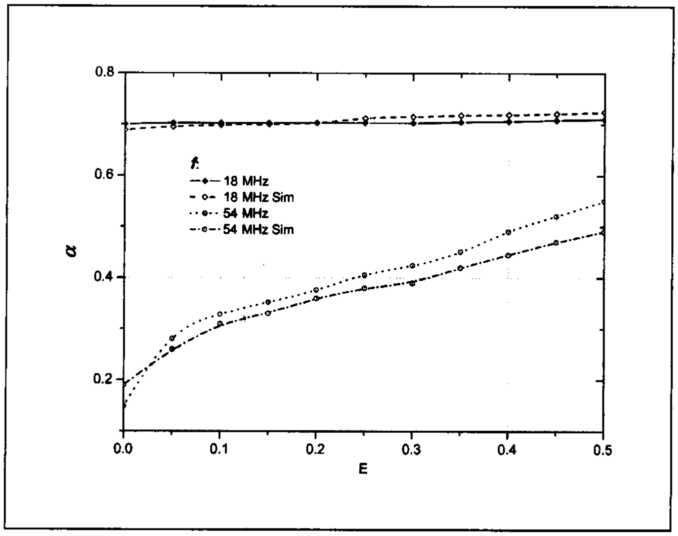

Figure 4 — Attenuation coefficient versus discontinuity density at different frequencies.

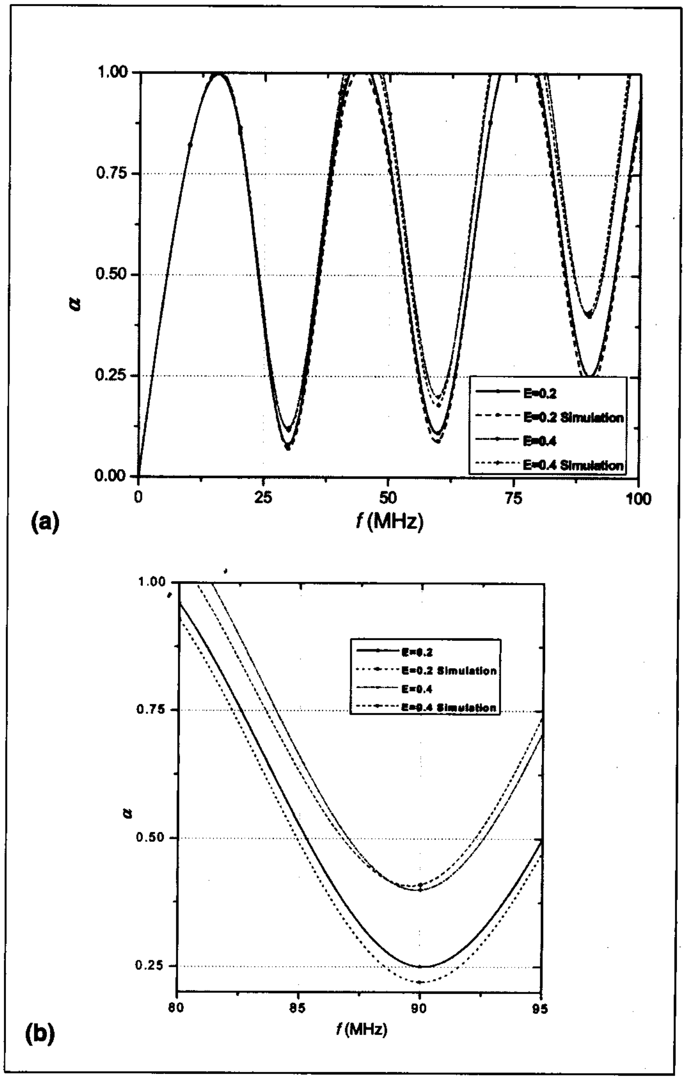

Figure 5 — Attenuation coefficient versus frequency for different discontinuity densities: (a) general view; (b) magnified view.

attenuation difference caused by changing crack densities becomes noticeable at higher frequencies, while the difference at low frequencies is not so obvious. We may conclude that if we want to use the attenuation change to distinguish crack density differences for NDT purposes, the frequency of wave used should be set relatively higher.

SUMMARY AND CONCLUSION

Ultrasonic waves propagating in a multilayer capacitor structure containing multiple cracks have been studied in this work. The reflections and transmissions at the layer interfaces were treated using the T-matrix technique, while effects of multiple cracks on wave propagation were evaluated using a scattering theory. Total velocity and attenuation changes for different frequencies and crack densities are calculated. Theoretical results have been further compared with finite element simulations. Good agreement has been reached in the frequency range of 10 to 200 MHz. Due to the linear shape of cracks and their random orientations, less attenuation of ultrasonic waves is observed than when transmitting through delaminations (Zhu and Cao, 2006). On the other hand, the results obtained herein indicate the potential of developing a high frequency ultrasonic NDT technique for evaluating small discontinuities in multilayer ceramic capacitors using the attenuation data.

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