Parameter Measurement of Thin Elastic Layers Using Low-Frequency Multi-Mode Ultrasonic Lamb Waves

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***Abstract*—A low-frequency ultrasonic method presented in this paper is suitable for measuring any one of the four characteristic parameters, i.e., thickness, density, shear, and longitudinal wave velocities, of a thin elastic layer given the remaining three. Thickness of the thin layer is much smaller than the longitudinal wavelength of the ultrasonic wave used in the experiments. The method employs the water immersion pitch–catch technique, and is based on the dispersion properties of the first-order symmetric and anti-symmetric modes of a Lamb waveguide. The unknown parameter is estimated by minimizing the mean square error obtained by comparing theoretical dispersion curves with experimental data. A wide-band excitation technique is used to reduce measurement time, by which a frequency range of possible guided Lamb waves is simultaneously explored. The sensitivity and accuracy of the proposed technique for different parameters in different thicknesses are analyzed. Using the present technique, a thin layer with thickness down to one percent of the longitudinal wavelength can be successfully measured with an average error of about two-percent. But the method fails to evaluate the density of the thin layer if the thickness is less than five percent of longitudinal wavelength.**

***Index Terms*—Dispersion, Lamb wave, nondestructive evaluation (NDE), thin layer, ultrasonics.**

I. Introduction

THERE are various situations of technological importance in which one wishes to carry out a nondestructive evaluation (NDE) of the characteristic parameters (i.e., thickness, density, shear, and longitudinal wave velocities) of thin layers, in order to evaluate the quality of welding seam, the strength of adhesive bonding layer in an adhesively-bonded joint, the thickness of protective surface coatings on substrate, and the layers in composite structures. A layer can be considered thin when its thickness is much smaller than the longitudinal wavelength of the probing ultrasonic wave. In this case, the conventional characterization methods, such as the pulse-echo method [1]–[3], the pulse interference [4] and the resonance testing method [5], cannot be applied. Although broadband high-frequency ultrasound could be used to characterize thin layers [6], it is very expensive. More importantly, high-frequency ultrasound has a very short penetration depth, which limits its application to only low loss materials. In addition, the high-frequency waves will suffer the interference from micro-structural details of the thin layers, such as individual plies, resin-rich regions and individual fibers.

Some low-frequency methods, such as the time-domain least square method [7], the frequency-domain transfer function method [8]–[11], and the variable trigger and strobe (VTS) method [12] have been proposed to characterize the thin layer with thickness down to one percent of the longitudinal wavelength. The time-domain least square method and the frequency-domain transfer function method have good performance for characterizing the thickness and density of thin layers, but the computationally convergence problem occurs if the layer thickness is less than one-half of the longitudinal wavelength. Moreover, the aforementioned methods cannot evaluate the shear wave velocity, which is an indispensable parameter for characterizing the mechanical properties of the thin layer.

Compared with conventional bulk waves, the Lamb waves are more appropriate for evaluating the characteristic parameters of thin layers since they include both the longitudinal and shear wave information. M. R. Karim *et al.* recently presented a method using Lamb waves to measure the characteristic parameters of layer materials with thickness ranging from a few micrometers to several meters [13]–[15]. They did not, however, evaluate the accuracy of their method and did not study the influence of thickness.

In this paper, a multi-mode Lamb wave method based on the detection of the first two guided Lamb modes is presented for the evaluation of one of the four parameters, thickness, density, shear, and longitudinal wave velocities of thin elastic layers when three of them are known. Sensitivity, accuracy for different parameters, and dependence of the presented method on the layer thickness are studied. The experimental results agree well with the theoretical calculations.

II. Methodology

A. Low-Frequency Multi-Mode Lamb Wave Method and Its Sensitivity Analysis

The properties of Lamb waves have been studied in detail [16]. The characteristic dispersion equations of the symmetric and anti-symmetric mode Lamb waves propagating in an isotropic elastic layer immersed in water are as follows

$$\begin{aligned} &F_s\left(v_l,\, v_s,\, h,\, \rho,\, v_L,\, \rho_L,\, f^s,\, v^s_{Lamb}\right) \\ &\quad = (k^2+s^2)^2 \mathrm{cth}\left(\frac{\mathrm{q\,h}}{2}\right) - 4k^2 qs\,\mathrm{cth}\left(\frac{\mathrm{s\,h}}{2}\right) \\ &\qquad - i\frac{\rho_L}{\rho}\,\frac{qk_s^4}{\sqrt{k_L^2-k^2}} = 0 \end{aligned} \tag{1}$$

Manuscript received October 4, 1999; revised May 18, 2001. This work was supported by the National Natural Science Foundation of China under Grant 69925101.

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Publisher Item Identifier S 0018-9456(01)08766-6.

$$F_a\,(v_l,\,v_s,\,h,\,\rho,\,v_L,\,\rho_L,\,f^a,\,v^a_{Lamb}) = \left(k^2+s^2\right)^2 \mathrm{th}\left(\frac{\mathrm{qh}}{2}\right) - 4k^2 qs\,\mathrm{th}\left(\frac{\mathrm{sh}}{2}\right) - i\frac{\rho_L}{\rho}\,\frac{qk_s^4}{\sqrt{k_L^2-k^2}} = 0. \tag{2}$$

Here

$$k=\frac{\omega}{v_{Lamb}}, \quad s=\sqrt{\left(\frac{\omega}{v_{Lamb}}\right)^2-\left(\frac{\omega}{v_s}\right)^2},$$

$$q=\sqrt{\left(\frac{\omega}{v_{Lamb}}\right)^2-\left(\frac{\omega}{v_l}\right)^2}, \quad k_s=\frac{\omega}{v_s}, \quad k_L=\frac{\omega}{v_L}$$

ω is the angular frequency of the ultrasonic wave. v_s, v_l, h, and ρ are the shear wave velocity, longitudinal wave velocity, thickness, and the density of the thin layer, respectively. ρ_L and v_L are the density and longitudinal ω_e velocity of water. $f^s(f^a)$ and $v^s_{Lamb}(v^a_{Lamb})$ denote the frequency and the velocity of a symmetric (anti-symmetric) mode Lamb wave, respectively.

Using the dispersion curves of the lowest order symmetric and anti-symmetric mode, i.e., the s_0 and a_0 mode Lamb waves, one can measure the parameters of a thin layer. One of the quantities $v_s, v_l, h,$ or ρ of the thin layer can be deduced through a comparison between the measured and the theoretically predicted dispersion curves. The value of $v_s, v_l, h,$ or ρ of the thin layer being calculated minimizes the residual error E_s or E_a defined below

$$\mathrm{E_s(p)}=\frac{1}{\mathrm{N}}\sum_{i=1}^{\mathrm{N}}\left|\mathrm{v}^{\mathrm{s}}_{Lamb,\,i}-\mathrm{v}^{*,\,\mathrm{s}}_{Lamb,\,i}\right|^2 \tag{3a}$$

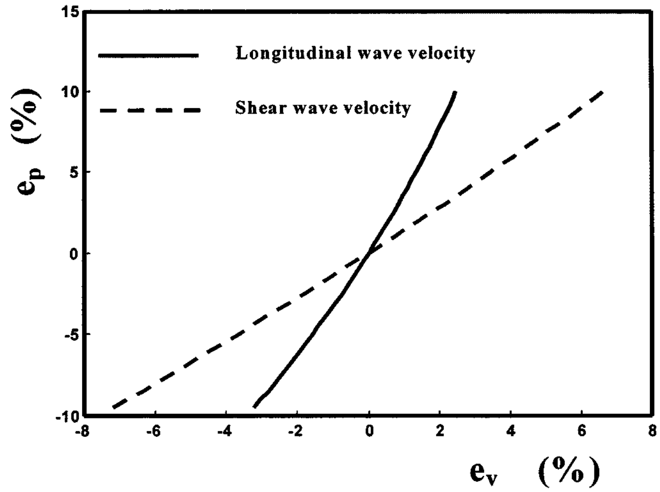
$$\mathrm{E_a(p)}=\frac{1}{\mathrm{M}}\sum_{j=1}^{\mathrm{M}}\left|\mathrm{v}^{\mathrm{a}}_{Lamb,\,j}-\mathrm{v}^{*,\,\mathrm{a}}_{Lamb,\,j}\right|^2 \tag{3b}$$

where "||" is the absolute value. p represents the value to be calculated, which can be v_s, v_l, h, or ρ. Only the measured velocities of Lamb waves are used to evaluate the layer parameters in this work. $(f_i^s,\,v^s_{Lamb,\,i})$, $i=1,2,\ldots,N$ and $(f_j^a,\,v^a_{Lamb,\,j})$, $j=1,2,\ldots,M$ are the set of experimental dispersion data for the s_0 and the a_0 Lamb waves in the phase velocity-frequency space, respectively. The value of M or N is directly related to the effective frequency range from 0.918 MHz∼3.463 MHz, in which the phase variation is linear. Besides the bandwidth of the frequency-thickness product, the angular resolution of the angle controller can also affect the value of M or N. Smaller angle resolution corresponds to larger values of M and N. $v^{*,\,s}_{Lamb,\,i}(v^{*,\,a}_{Lamb,\,i})$ is the theoretical Lamb wave velocity of the first-order symmetric (anti-symmetric) Lamb wave calculated from the characteristic dispersion equation. For convenience, we name the characterization technique as the s_0-method when the dispersion curve of s_0-Lamb wave is used, and the a_0-method when the a_0 Lamb wave is used.

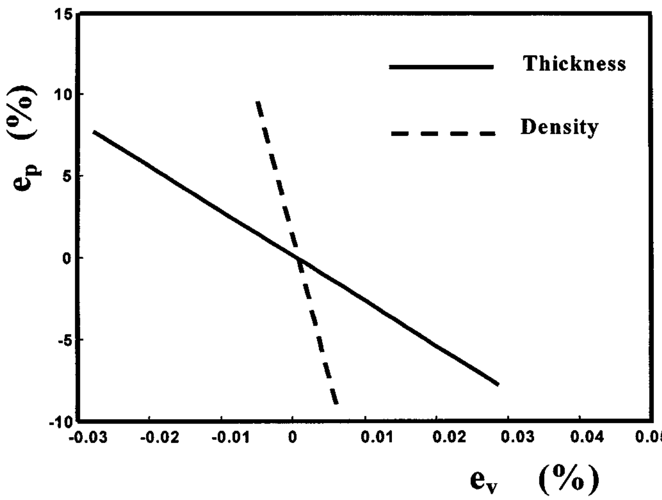
To obtain the value of a parameter p, we differentiate equations (3a) and (3b) with respect to p. The optimum value of p should satisfy the equations below

$$\frac{\partial \mathrm{E_s}(p)}{\partial p}=0 \tag{4a}$$

$$\frac{\partial \mathrm{E_a}(p)}{\partial p}=0. \tag{4b}$$



(a)



(b)

Fig. 1. The e_p–e_v curve of (a) the longitudinal and shear velocities and (b) thickness and density using the s_0-method (material: Aluminum, $v_l = 6.206$ mm/μs, $v_s = 3.045$ mm/μs, $h = 26$ μm).

Substituting (3) into (4) and employing the Taylor's expansion of $\mathrm{v}^*_{\mathrm{Lamb,\,i}}$ at $\mathrm{p}=\mathrm{p}^*$, where p^* is the estimation value of p, it can be derived that the relative error e_p is given by

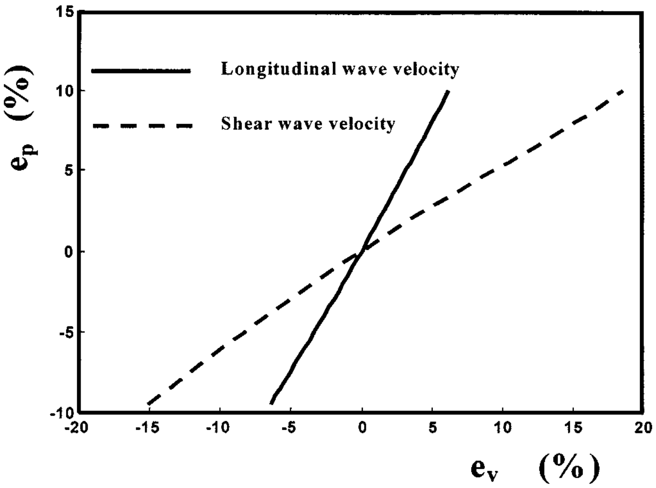
$$\mathrm{e_p}=\frac{\sum_{i=1}^{\mathrm{K}}\mathrm{e}_{v_i}\cdot \mathrm{S}_{v_i,\,p}\cdot(\mathrm{v}_{Lamb,\,i})^2}{\sum_{\mathrm{i=1}}^{\mathrm{K}}\mathrm{S}^2_{v_i,\,p}\cdot(\mathrm{v}_{Lamb,\,i})^2} \qquad (\mathrm{K=M,\,N}) \tag{5}$$

where the Taylor's expansion just preserves the linear term of absolute error of p, $\mathrm{e_{v_i}}$ is the relative error of $\mathrm{v_{Lamb,\,i}}$, $S_{v,p}$ is the sensitivity of v_{Lamb} to the variation of p,

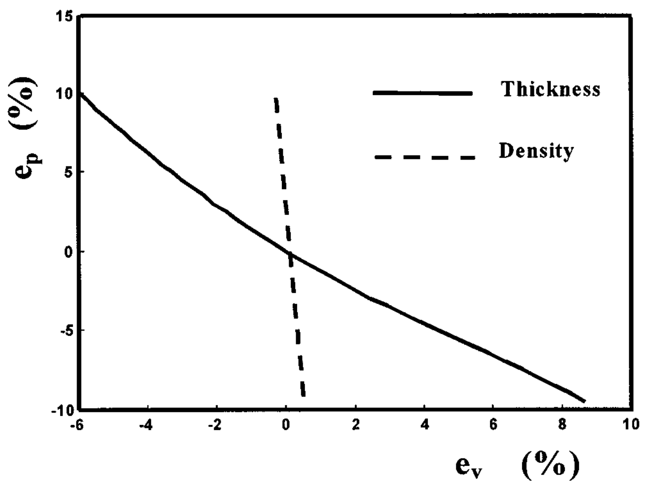
$$S_{v_i,p}=\frac{\Delta v_{Lamb}/v_{lamb}}{\Delta p/p} \qquad (i=1,\ldots,M \text{ or } N). \tag{6}$$

From (6), if the sensitivity is small, a small error in measuring v_{Lamb} will result in a large error in estimating p.

Shown in the Fig. 1 are the theoretical curves of the relative error of the Lamb wave velocity e_p versus e_v when a 26 μm thick aluminum layer is characterized by the s_0-method using a pair

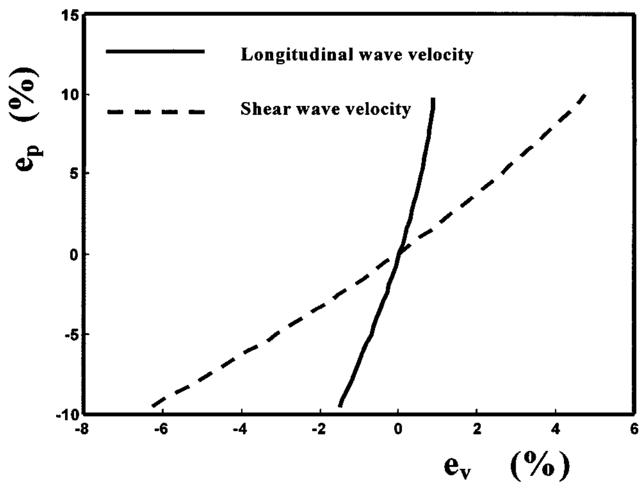


(a)

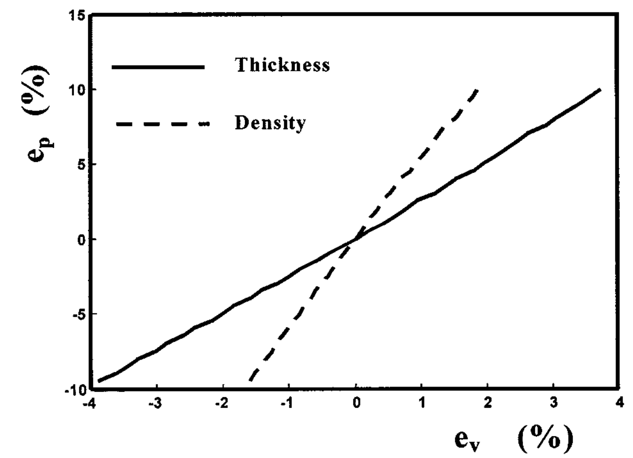


(b)

Fig. 2. The e_p–e_v curve of (a) the longitudinal and shear velocities and (b) thickness and density using the s_0-method (material: Aluminum, $v_l = 6.209$ mm/μs, $v_s = 3.045$ mm/μs, $h = 521$ μm).



(a)



(b)

Fig. 3. The e_p–e_v curve of (a) the longitudinal and shear velocities and (b) thickness and density using the a_0-mode Lamb wave method (material: Aluminum, $v_l = 6.207$ mm/μs, $v_s = 3.048$ mm/μs, $h = 152$ μm).

of broadband transducers with 2 MHz center frequency on the condition of $\mathrm{N} = 1$. It can be seen from Fig. 1(a) that the relative error of v_s is slightly smaller than that of v_l with the same v^s_{Lamb} measurement error. Fig. 1(b) shows that the s_0-method has very poor sensitivity for measuring thickness and density of the same aluminum thin layer. In other words, the errors of h and ρ are much larger than those of v_s and v_l at the same level of e_v. This phenomenon is further substantiated by experiments.

We also theoretically calculated the e_p–e_v curve for aluminum layers of different thicknesses in order to find the limitation of the s_0-method. Shown in Fig. 2 is the theoretical e_p–e_v curve for an aluminum layer with thickness of 521 μm using the same 2 MHz center frequency transducers on the condition of $\mathrm{N} = 1$. It is found that the sensitivities of the s_0 method to different parameters increase as the layer thickness increases but at different rates. The sensitivity to h increases very fast so that it will be larger than those of v_s and v_l at some thickness values.

The sensitivities of the a_0-method to all the parameters have also been studied. Providing $\mathrm{M} = 1$, the e_p–e_v curves corresponding to all parameters are shown in Fig. 3 when using the a_0-method to characterize a 152 μm aluminum layer. It can be seen that the sensitivity of the a_0-method to v_s or v_l is slightly smaller than that of the s_0-method as a whole. On the other hand, the sensitivities of the a_0-method to h and ρ are much larger than that of the s_0-method.

From the sensitivity analysis above, we conclude that v_s and v_l of the thin layer can be evaluated by the s_0-method, while the a_0-method is more suitable to characterize h and ρ.

B. Inversion Algorithm

Compared with the steepest descend and Newton–Raphson methods, or a combination of the two such as the Marquardt algorithm [17], the simplex algorithm [16] has been verified to be the most appropriate for solving the nonlinear inversion problem because of its good convergence property. In this paper, we have used the simplex algorithm for curve fitting and finding the roots of (3) and (4). The effective wavenumber $k_{s,\,a}$ is complex due to the fact that the Lamb wave leaks its energy into the water along the path of propagation. The real and imaginary parts of $k_{s,\,a}$ denote the wavenumber and the attenuation of the Lamb wave, respectively. In our problem, the total number of variables to be

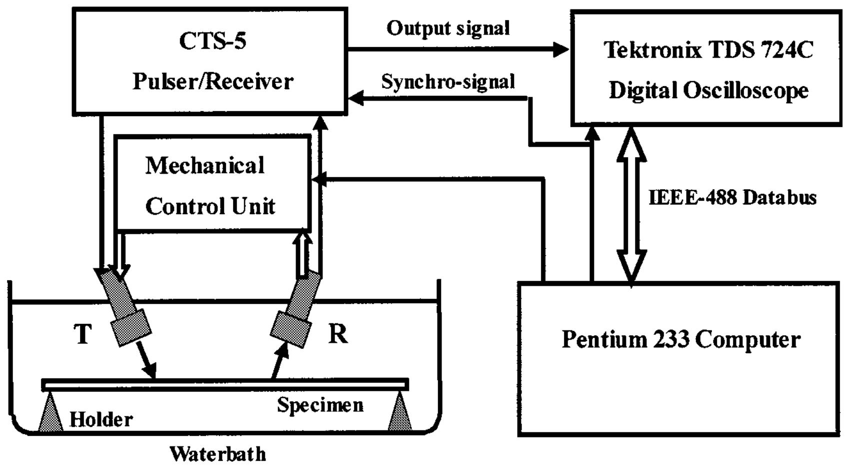


Fig. 4. Block diagram of experimental setup.

optimized is two. One is a parameter among v_s, v_l, h, and ρ, and the other is the imaginary part of $k_{s,a}$.

III. Equipment and Experimental Procedure

A schematic diagram of the experimental apparatus for the low-frequency multi-mode Lamb wave method is shown in Fig. 4. A pair of accurately matched broadband water immersion type longitudinal wave transducers, T and R, with a center frequency of 2 MHz, was used for sending and receiving the ultrasonic waves. The transmitting and receiving angles are adjusted by the Mechanical Control Unit to maximize the signal. The experimental process can be described as follows: A pulse of 5 ns duration produced by the CTS-5 Pulser/Receiver is sent to the transmitter, which emits broadband pulse ultrasound to the specimen. Lamb waves are produced and propagate along the thin layer. At the other end of the layer material, leaky Lamb signals are received and amplified by a CTS-5 Pulser/Receiver, then, digitized at a preset sampling rate of 100 MHz by a Tektronix digital oscilloscope. To reduce the random error, each signal is averaged 100 times before it is transferred to a computer through a GPIB (IEEE-488) databus for further analysis.

The relative distance between the two transducers and the distance from the transducer to specimen surface are set to specific values based on geometric calculations considering beam diffraction [18]. For the optimum setting, only the leaky Lamb wave signals are received by the receiver. The effects of ultrasonic waves reflected between the layer surface and the water surface, and between the layer surface and the bottom of the water tank, and waves propagating through the water directly from the transmitter to the receiver, can all be eliminated. The geometric setting is illustrated in Fig. 5.

In Fig. 5, θ denotes the incidence angle, W is the distance between the transmitter and the receiver, and H is the distance from the transducer to the surface of specimen. All those three parameters can be controlled by the Mechanical Controlled Unit. The value β is half of the angle spanned by the main lobe, which is decided by the equation given below [18]

$$\beta = \arcsin(1.2\lambda/D) \tag{7}$$

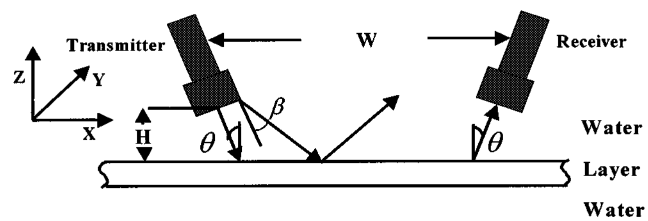


Fig. 5. Sketch of the geometric arrangement of sample and the transducers.

where D is the diameter of the transducers and λ is the wavelength of incidence ultrasound in water. The transducer-specimen distance H is adjusted based on the equation below for the specimen in the far-field of the transducer

$$H \geq (D^2 - \lambda^2)\cos(\theta)/4\lambda. \tag{8}$$

Only when W satisfies the relation below can the leaky Lamb wave signals be received by the receiver (omitting the dimension of the transducers)

$$W \geq 2H/\tan(\theta + \beta). \tag{9}$$

Using the Fourier transform, the received leaky Lamb wave pulse signals can be used to derive the dispersion curves in the elastic layers. Peaks are present in the amplitude spectrum indicating the presence of Lamb wave roots. The phase velocities for the Lamb waves are selected by controlling θ, which is adjusted by the Mechanical Control Unit. The value of the Lamb wave phase velocity can be calculated through the Snell's law

$$v_{Lamb} = v_L/\sin(\theta) \tag{10}$$

where v_L denotes the longitudinal ultrasound velocity in water, which is taken as 1.49 mm/μs in our experiments. The experiments are conducted under a constant temperature of 20 °C. The measurement of the Lamb roots is repeated for a set of angles θ_j, i.e., for a set of phase velocity values. The mechanical structure of the transducer holder permits the incident angle to be adjusted ranging from 14° to 54° with an angle resolution of 15′, which corresponds to a range of 1800 m/s to 6000 m/s for the phase velocities of Lamb waves to be detected. The transmitted and received pulses are shown in Fig. 6 both in time and frequency domains.

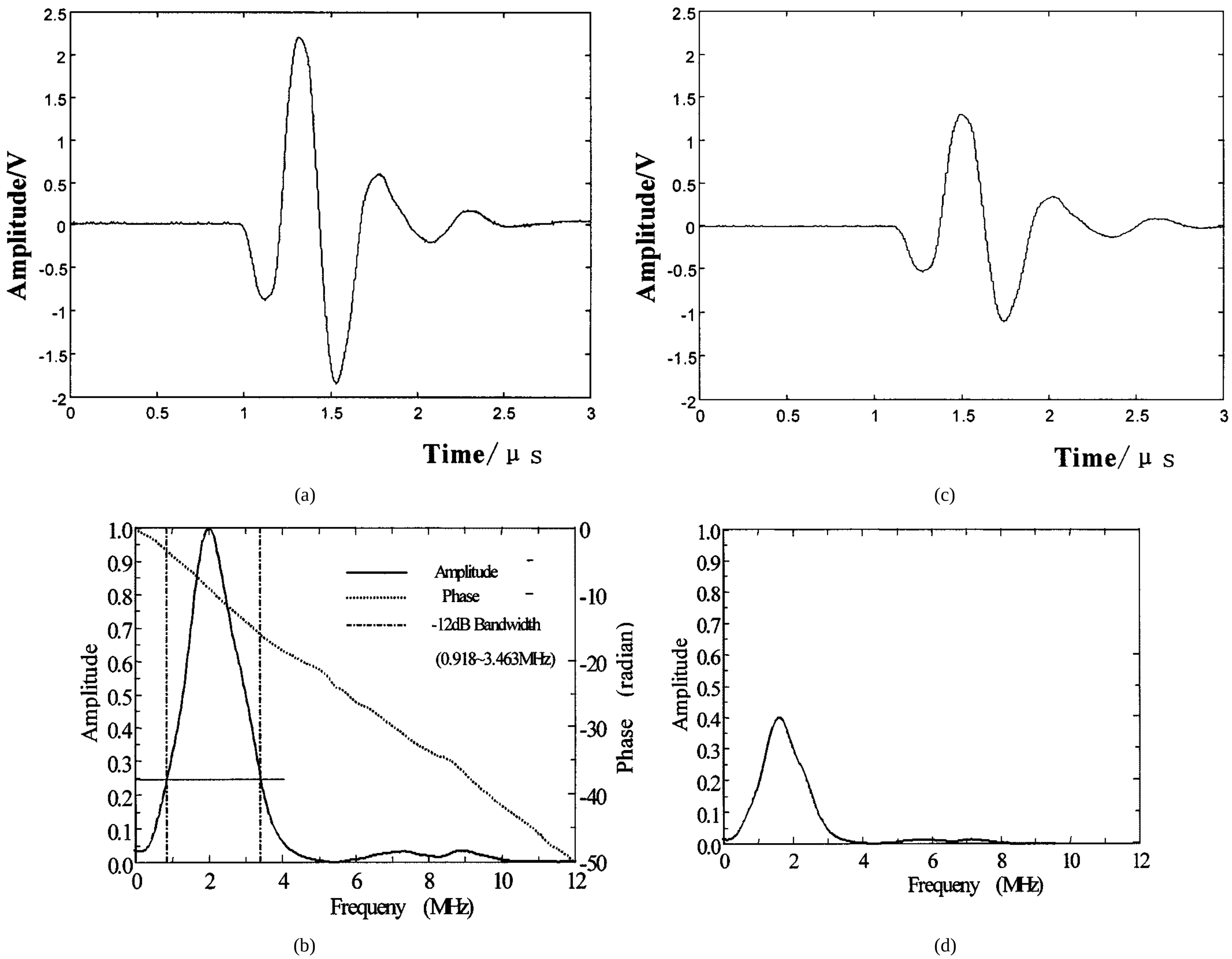


Fig. 6. Waveforms of (a) the transmitted and (c) received pulses, and the amplitude spectrums of (b) the transmitted and (d) received pulses.

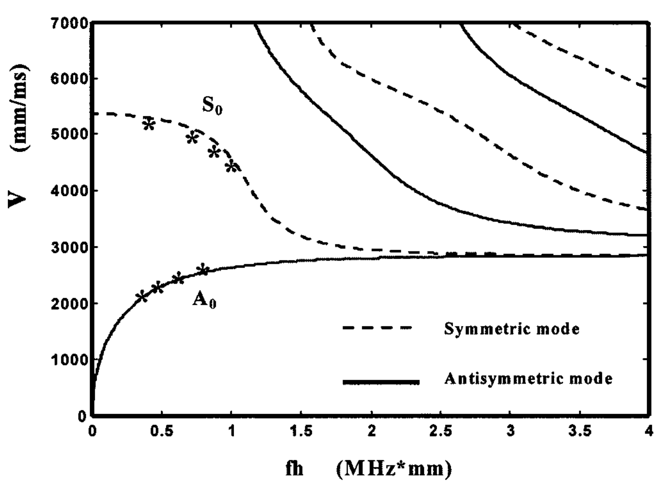


Fig. 7. Dispersion of the symmetric and anti-symmetric Lamb waves, and the experimental data of a thin aluminum layer ($v_l = 6.210$ mm/μs, $v_s = 3.047$ mm/μs, $h = 256$ μm).

IV. Results and Discussions

Denoted by the star "*," the experimental dispersion data for an aluminum layer with thickness of 256 μm is shown in Fig. 7, which is obtained using our method with a pair of 2MHz center frequency broadband transducers.

The main objective of this work is to develop a technique suitable for characterizing the shear wave velocity, the longitudinal wave velocity, the thickness, and the density of thin elastic layers of only a few percent of the longitudinal wavelength in thickness. A number of specimens made of aluminum, iron, brass, and glass with different thicknesses have been measured using the newly developed low-frequency multi-mode Lamb wave method. The standard values of the thickness and the densities of the specimens are tested by the electronic micrometer and the Archimedes' theory, respectively, and the standard values of the shear wave and longitudinal wave velocities of the specimens are measured by the VTS method using high-frequency longitudinal and shear wave transducers [12]. The measured results using the low-frequency multi-mode Lamb wave method are presented in Tables I and II, in which the blanks represent the evaluation errors that are larger than 10% and the corresponding measured value is omitted. The standard parameter values are listed under the symbols $\hat{h}$, $\hat{\rho}$, $\hat{v}_l$, and $\hat{v}_s$.

In order to compare with those low-frequency ultrasound methods mentioned in the introduction, the capabilities of

TABLE I
CALCULATED PARAMETERS FOR THIN ELASTIC LAYERS OF DIFFERENT THICKNESS USING THE s_0-METHOD

Specimen Material	$\hat{h}$	$\hat{\rho}$	$\hat{v}_l$	$\hat{v}_s$	h		ρ		v_l		v_s	
	μm	g/mm³	mm/μs	mm/μs	μm	Err(%)	g/mm³	Err(%)	mm/μs	Err(%)	mm/μs	Err(%)
1 Aluminum	26	2.69	6.206	3.045	/	/	/	/	6.060	-2.4	3.011	-1.1
2 Aluminum	112	2.69	6.201	3.044	/	/	/	/	6.068	-2.1	3.015	-1.0
3 Aluminum	256	2.68	6.210	3.047	264	3.3	/	/	6.079	-2.1	3.024	-0.7
4 Aluminum	521	2.69	6.209	3.045	529	1.5	2.81	4.4	6.077	-2.1	3.019	-0.1
5 Iron	76	7.86	5.949	3.241	/	/	/	/	5.880	-1.1	3.219	-0.1
6 Brass	101	8.47	4.714	2.093	/	/	/	/	4.797	1.8	2.103	0.5
7 Glass	115	2.32	5.579	3.319	/	/	/	/	5.574	-0.1	3.313	-0.2

TABLE II
CALCULATED PARAMETERS FOR THIN ELASTIC LAYERS OF DIFFERENT THICKNESS USING THE a_0-METHOD

Specimen	Material	$\hat{h}$	$\hat{\rho}$	$\hat{v}_l$	$\hat{v}_s$	h		ρ	
		μm	g/mm³	mm/μs	mm/μs	μm	Err(%)	g/mm³	Err(%)
1	Aluminum	152	2.69	6.207	3.048	156	2.6	/	/
2	Aluminum	256	2.68	6.210	3.047	252	-1.6	2.89	7.5
3	Iron	106	7.85	5.941	3.238	105	-0.1	7.96	1.5
4	Brass	381	8.48	4.711	2.090	382	0.3	8.35	-1.5
5	Glass	216	2.33	5.578	3.317	217	0.5	2.27	-2.5

the s_0- and a_0-method for the layer thickness measurement are evaluated using the minimum ratio of h/λ_l. As shown in Table I, the low-frequency s_0-method has successfully evaluated v_s and v_l of thin aluminum layers with thickness ranging from 26 μm to 512 μm, i.e., 1% λ_l to 16% λ_l, where $\lambda_l = 31.05$ μm, with the evaluation errors smaller than 3%. The evaluation errors are found to be thickness dependent and become progressively worse as thickness decreases. On the other hand, for thickness characterization, the lowest layer thickness limit of the s_0-method for aluminum has been proved to be about 300 μm, i.e., 10% of λ_l.

The main cause of the evaluation errors is attributed to the measurement error of the Lamb wave velocity, which has a direct relation with the angle accuracy of the transmitter and receiver. On the condition that the angle resolution is 15′, the possible measurement errors of the s_0 mode Lamb wave velocities at different incidence angles are shown in Fig. 8. It can be noticed that the measurement error of the Lamb wave velocities decreases as the incidence angle increases (corresponding to the decrease of Lamb wave velocity). Since the s_0 mode Lamb wave velocity decreases with the layer thickness, the possible measurement error of the Lamb wave velocity also decreases, which is another reason for the improvement of the accuracy for v_s and v_l besides the increased sensitivities. Moreover, it can be shown that the s_0-method is more appropriate to measure the specimens with lower acoustic wave velocity, which has been verified by the experimental results shown in Table I.

From Table II, it is shown that the a_0-method can be used to evaluate the thickness and density of thin layers with thickness of about 5% λ_l. The evaluation errors are smaller than 3% except for evaluating the aluminum density. Due to the water-coupled mode in our experiment, the a_0-method fails when the phase velocity is smaller than the velocity of ultrasound in water, which sets the layer thickness limit of the a_0-method.

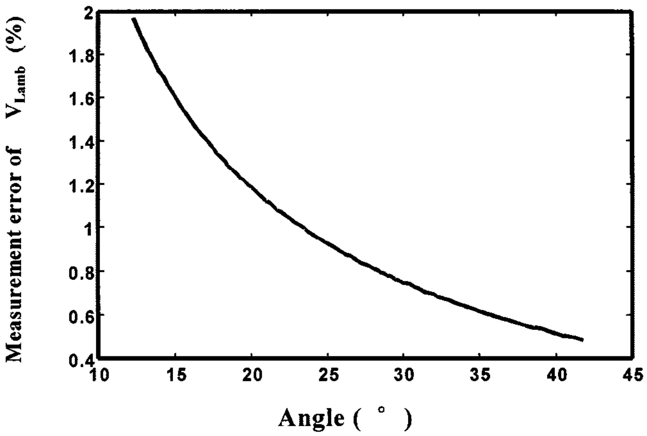


Fig. 8. Possible measurement errors of the s_0 mode Lamb wave velocities at different incidence angles with the angle resolution of 15′.

V. Conclusion

A low-frequency multi-mode ultrasonic Lamb wave method is developed to measure any one of the four acoustic parameters, thickness, density, shear, and longitudinal wave velocities, of thin elastic layers given the other three parameters. The method has been proved to be accurate for specimens with thickness down to one percent of the longitudinal wavelength. The average evaluation error for v_s or v_l using the a_0-method is about two percent. In order to resolve the relatively poor sensitivity of the s_0-method to the evaluation of the thickness and density, one can use the a_0-method. The h/λ_l limitation for the a_0-method is about five percent of λ_l and the evaluation error is smaller than three percent.

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