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Low frequency ultrasonic multi-mode Lamb wave method for characterizing the ultra-thin transversely isotropic laminate composite: Theory and experiment *

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Abstract A low-frequency multi-mode ultrasonic Lamb wave method suitable for characterizing the thickness, the density and the elastic constants of the ultra-thin transversely isotropic laminate composite is presented. The "ultra-thin" here means that the thickness of the plate is much less than the wavelength of the ultrasonic wave so that the echoes from the front and back faces of the plate can't be separated in the time domain. The dispersion equations for the low frequency ultrasonic Lamb waves with the propagation directions parallel and vertical to the fiber direction are derived. In conjunction with the least square algorithm method, the secant algorithm is used to estimate the parameters of the ultra-thin fiber-reinforced composite layer. The evaluation errors and the sensitivity of the method to different parameters of the thin composite are analyzed. The technique has been used to characterize the ultra-thin grass fiber reinforced PES composite with thickness down to ten percents of the ultrasonic wavelength. It is observed that the agreement between the nominal and the estimation values is reasonably good.

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1 Introduction

The transversely isotropic laminate composites, such as the unidirectional fiber-reinforced composites and the unidirectional stretched high polymers, are widely used in many fields such as aerospace, information science and biomedical engineering. Recently, more and more attention has been attracted to quantitative, nondestructive and in-situ evaluating of the acoustical, mechanical and interfacial characters of the transversely isotropic laminate composites.

The traditional ultrasonic pulse-echo method failed to characterize the thin transversely isotropic laminate composites. The "thin" here means that the thickness of the plate is only a

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** W. Cao is with the Intercollege Materials Research Laboratory, The Pennsylvania State University, University Park, PA, 16802, USA fraction of the ultrasonic wavelength so that the echoes from the front and back surfaces of the plate can't be separated in the time domain. Although the single-pulse ultrasonic transducer with boarder band and higher frequency can be used to measure the thinner sample, we prefer to use low-frequency ultrasound for two reasons. For one thing, the use of such high frequencies not only greatly increases the cost of the transducer and associated broadband instrumentation, but also limits its application because of very short penetration depth. For another, the scattering caused by microstructure details such as plies, resin-rich regions between plies and individual fibers would also interferes with the characterization process.

Compared with the ultrasonic bulk waves, the ultrasonic Lamb waves are much proper to evaluate the parameters of the thin composite. They carry more information of acoustical, mechanical and interfacial properties about the thin composite because of their multi-mode, dispersion properties, which are directly influenced by the sound velocity, the thickness, the density, the elastic constants, the fiber direction of the thin composite, etc. In this paper, a low-frequency multi-mode ultrasonic method suitable for characterizing thickness, density and elastic constants of the ultra-thin transversely isotropic laminate composite is presented, in which the dispersion properties of the low frequency Lamb waves in the directions parallel or vertical to the fiber direction is used. In conjunction with the method of the least square, the secant algorithm is used to estimate the parameters of the ultra-thin fiber-reinforced composite layer. The evaluation errors of the parameters and the sensitivity of the method to different parameters are analyzed. Reasonably good comparison between the theory and experiment is observed.

2 Method

2.1 Low-frequency multi-mode ultrasonic Lamb wave method

The elastic characteristics of the ultra-thin transversely isotropic laminate composite are the same as the hexangular symmetrical crystal, which has five independent elastic constants: C_{11}, C_{12}, C_{13}, C_{33} and C_{44}. Supposing Z-axis is the fiber direction of the composite or the unidirectional stretching direction of the high polymer and X-axis is perpendicular to the surface of the ultra-thin layer, the relationship between the stress σ_{ij} and the strain ε_{ij} of the ultra-thin transversely isotropic laminate composite can be expressed as

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{13} & 0 & 0 & 0 \\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \varepsilon_{23} \\ \varepsilon_{13} \\ \varepsilon_{12} \end{bmatrix}, \tag{1}$$

where $C_{66} = (C_{11} - C_{22})/2$.

The strain-displacement relation can be written as

$$\varepsilon_{ij} = \frac{U_{i,j} + U_{j,i}}{2}, \qquad i, j = 1, 2, 3. \tag{2}$$

The motion equation of the elastic medium is

$$\sum_{j=1}^{3} \sigma_{ij,j} = \rho \ddot{U}_i \qquad i = 1,2,3. \tag{3}$$

where ρ is the density of the laminate composite. Since the displacement gradient in the direction of Y-axis is zero, the displacement in the direction of X-axis and Z-axis may be supposed as following forms when the ultrasonic Lamb wave is propagating along Z-axis direction

$$U_1 = U_{10} e^{[i(k_x x + k_z z - \varpi t)]}, \tag{4}$$

$$U_3 = U_{30} e^{[i(k_x x + k_z z - \varpi t)]}, \tag{5}$$

where k_x and k_z are the wave number components of the Lamb wave in the X-axis and Z-axis directions. $k_z = \varpi / v_z$, $\varpi = 2\pi f$, where f is the frequency of the ultrasonic wave, v_z is the propagation velocity of Lamb wave in the Z-axis direction. Substituting Eqs. (1), (2), (4) and (5) into (3), one obtains

$$\rho \varpi^2 U_{10} = C_{11} U_{10} k_x^2 + (C_{13} + C_{44}) U_{30} k_x k_z + C_{44} U_{10} k_z^2, \tag{6}$$

$$\rho \varpi^2 U_{30} = C_{44} U_{30} k_x^2 + (C_{13} + C_{44}) U_{10} k_x k_z + C_{33} U_{30} k_z^2. \tag{7}$$

Eliminating U_{10} and U_{30} from Eqs. (6) and (7), we get a quadratic equation for k_x in terms of k_z and the ratio of U_{10} to U_{30}

$$R_p = \left(\frac{U_{10}}{U_{30}} \right)_p = \frac{(C_{13} + C_{44}) k_{xp} k_z}{\rho \varpi - C_{11} k_{xp}^2 - C_{44} k_z^2}. \tag{8}$$

Let's defines k_{xp} and k_{xm} as the two roots of k_x calculated from Eqs. (6) and (7), and R_p and R_m are the values of R when k_x is k_{xp} and k_{xm} respectively. The two possible partial displacements are

$$U_1 = e^{[i(k_z z - \varpi t)]} \left\lfloor M e^{ik_{xp} x} + N e^{-ik_{xp} x} + P e^{ik_{xm} x} + Q e^{-ik_{xm} x} \right\rfloor, \tag{9}$$

$$U_3 = e^{[i(k_z z - \varpi t)]} \left\lfloor R_p (M e^{ik_{xp} x} - N e^{-ik_{xp} x}) + R_m (P e^{ik_{xm} x} - Q e^{-ik_{xm} x}) \right\rfloor. \tag{10}$$

when $M = -N$ and $P = -Q$, Eqs. (9) and (10) express the displacement of the symmetrical Lamb wave. Otherwise, when $M = N$ and $P = Q$, Eqs. (9) and (10) express the displacement of the anti-symmetrical Lamb wave. Moreover, the four boundary conditions must be satisfied: the normal stress in the plate and shear stress on the plate are zero on both surfaces of the thin layer, $z = \pm d/2$. Substituting Eqs. (9) and (10) into the four boundary conditions, the following set of equations is obtained

$$\begin{bmatrix} G_p V_p & G_p / V_p & G_m J_p & G_m J_p \\ G_p / V_p & G_p V_p & G_m J_p & G_m J_p \\ H_p V_p & -H_p / V_p & H_m J_p & -H_m / J_p \\ H_p / V_p & -H_p V_p & H_m / J_p & -H_m J_p \end{bmatrix} \begin{bmatrix} M \\ N \\ P \\ Q \end{bmatrix} = 0, \tag{11}$$

where $G_p = C_{11} R_p k_{xp} + C_{13} k_z$, $H_p = k_{xp} + k_z R_p$, $G_m = C_{11} R_m k_{xm} + C_{13} k_z$, $H_m = k_{xm} + k_z R_m$. For a nontrivial solution, the 4×4 determinant above in Eq. (11) must vanish. So, we get the dispersion equations for symmetrical and anti-symmetrical Lamb waves as the following equation

$$f_z(\rho, d, C_{11}, C_{13}, C_{33}, C_{44}, f, v_z) = \frac{\mathrm{tg}(k_{xp}d/2)}{\mathrm{tg}(k_{xm}d/2)} - \left[\frac{G_pH_m}{G_mH_p}\right]^{\pm 1} = 0. \tag{12}$$

where the symbol '+' and '−' correspond to the symmetrical and the anti-symmetrical Lamb wave respectively.

Similarly, when the propagation direction of the ultrasonic Lamb wave is parallel to Y-axis, the dispersion equation of the ultrasonic Lamb wave propagating in the ultra-thin transversely isotropic laminate composite can be deduced to the following equation

$$f_Y(\rho, d, C_{11}, C_{12}, f, v_y) = \frac{\mathrm{tg}(k_{xp}d/2)}{\mathrm{tg}(k_{xm}d/2)} - \left[\frac{G_pH_m}{G_mH_p}\right]^{\pm 1} = 0, \tag{13}$$

the meaning of the symbol '+' and '−' is the same as the Eq. (13), while R_p and R_m in Eq. (13) are expressed as the following

$$R_p = \frac{(C_{12} + C_{66})k_{xp}k_y}{\rho\varpi^2 - C_{11}k_{xp}^2 - C_{66}k_y^2}, \quad R_m = \frac{(C_{12} + C_{66})k_{xm}k_y}{\rho\varpi^2 - C_{11}k_{xm}^2 - C_{66}k_y^2}, \tag{14}$$

where k_y is the wave number component of Lamb wave in the y direction, $k_y = \omega/v_y$, v_y is the propagation velocity of the Lamb wave in the y direction. We get a quadratic equation for k_x in terms of k_y

$$(C_{12} + C_{66})^2 k_x^2 k_y^2 = (\rho\varpi^2 - C_{11}k_y^2 - C_{66}k_x^2)(\rho\varpi^2 - C_{11}k_x^2 - C_{66}k_y^2). \tag{15}$$

from which the two roots of k_x can be obtained and are denoted as k_{xp} and k_{xm}.

By using the method of the least square error, the evaluation result of the thin composite layer parameters is deduced through a comparison between the measured and the theoretically predicted dispersion curves of the Lamb waves, i.e., the evaluation result of the thin composite layer parameters should minimize the residual error $E(p)$ defined as

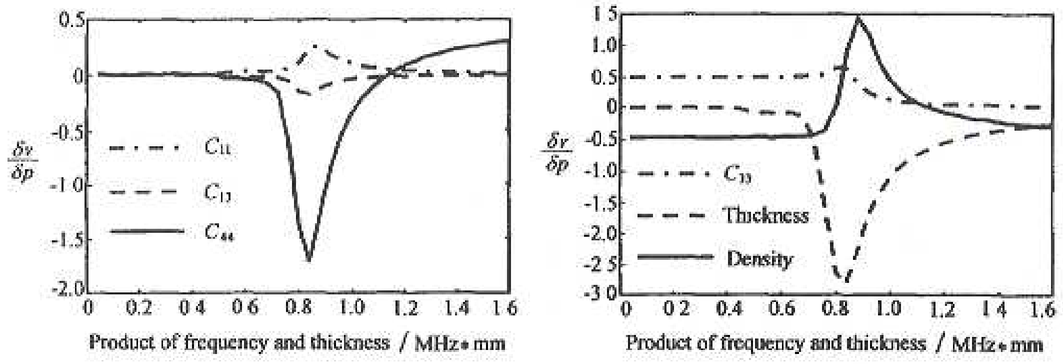
$$E(p) = \frac{1}{N}\sum_{j=1}^{N} \| f_z(\rho, d, C_{11}, C_{13}, C_{33}, C_{44}, f, v_z) \|^2, \tag{16}$$

$$E(p) = \frac{1}{N}\sum_{j=1}^{N} \| f_Y(\rho, d, C_{11}, C_{12}, f, v_y) \|^2, \tag{17}$$

where p is any one of ρ, d, C_{11}, C_{12}, C_{13}, C_{33} and C_{44}, N is the number of experimental dispersion data of the Lamb wave in the phase velocity-frequency space. The material parameter is deduced by the secant method which minimize the $E(p)$ in Eqs. (16) and (17). The initial values for the iterating procedure are the estimating values for the material parameters, which are in the range of ±30% around the real values.

2.2 Sensitivity analysis

It is clear that both the Lamb waves propagating in the direction of Z-axis and Y-axis are related to the density, thickness and C_{11} of the ultra-thin transversely isotropic laminate composite. However, the other four constants C_{12}, C_{13}, C_{33} and C_{44} are related to the Lamb wave propagating in the direction of Z-axis or Y-axis. The measurement errors on the Lamb wave velocity are the main cause for the evaluation error of the thin composite layer parameters. The parameter evaluation accuracy is dependent on the sensitivity of the methods to every parameter. Because the dispersion equation of Lamb wave is a transcendental function and it is difficult to directly get partial derivative of the Lamb wave velocity to every parameter, we calculate the relative change of the Lamb wave velocity caused by the one percent increment of every parameter. It is shown in Fig. 1 the sensitivity of ρ, d, C_{11}, C_{13}, C_{33} and C_{44} by the method using the symmetrical Lamb wave propagating in the direction of Z-axis.



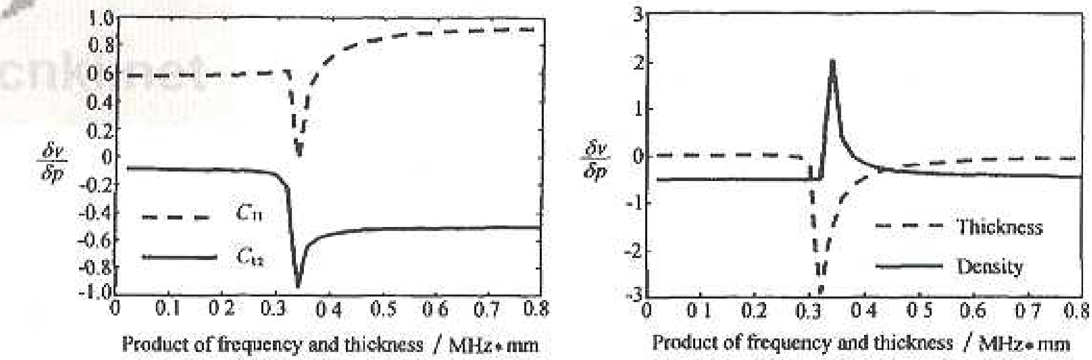
(a) Sensitivity for C_{11}, C_{13}, C_{44} (b) Sensitivity for C_{33}, thickness and density

Fig. 1 The sensitivity of ρ, d, C_{11}, C_{13}, C_{33} and C_{44} by the method using the symmetrical Lamb wave propagating in the direction of Z-axis

(Material: The unidirectional glass fiber enhanced polyethersulfone (PES))

It can be learned from the Fig. 1 that the sensitivity of the method using the symmetrical Lamb wave propagating in the direction of Z-axis depends on the thickness of the ultra-thin transversely isotropic laminate composite, but it isn't proportional to the thickness. In other words, the sensitivity doesn't augment in company with the material's thickness. In fact, the sensitivity of the method almost decreases to zero as the material's thickness increases to a certain values, because of the transit from the Lamb wave to the Rayleigh surface wave as the composite becomes thicker.

It is shown in Fig. 2 that the sensitivity of ρ, d, C_{11} and C_{12} by the method using the symmetrical Lamb wave propagating in the direction of Y-axis. From Fig. 1 and Fig. 2 we learn that the best product of the frequency and the thickness for the sensitivities of the Z-axis and Y-axis methods to ρ, d of the thin composite layer are different. The sensitivity of C_{11} is better by the Y-axis method. So, we can draw a conclusion that the symmetrical Lamb wave propagating in the direction of Z-axis is proper to characterize ρ, d, C_{13}, C_{33}, and C_{44}, while the symmetrical Lamb wave propagating in the direction of Y-axis is fit to characterize ρ, d, C_{11} and C_{12}.



(a) Sensitivity for C_{11}, C_{12} (b) Sensitivity for thickness and density

Fig. 2 The sensitivity of ρ, d, C_{11} and C_{12} by the method using the symmetrical Lamb wave propagating in the direction of Y-axis

(Material: The unidirectional glass fiber enhanced polyethersulfone (PES))

3 Experiment

3.1 Experimental system

A schematic diagram of the experimental apparatus for the low-frequency multi-mode Lamb wave method is shown in Fig. 3. A pair of accurately matched broadband longitudinal wave transducers, T and R, with a center frequency of 2 MHz, were used for generating and receiving the ultrasonic waves. The optimum transmitting and receiving angles are adjusted by the mechanical control unit and maintain equal to each other. The transducers and the measured material are coupled by the ultrasonic couplant. The angle resolution is 0.5°. A computer controls the general experimental procedure. A pulse of 5 ns duration produced by the CTS-5 Pulser/Receiver is sent to the transmitter which produces the broadband ultrasound pulse to the specimen. The leaky Lamb signals are received and amplified by the CTS-5 Pulser/Receiver, then digitized at a sampling rate 100 MHz by Tektronix digital oscilloscope. To reduce the random error, each test is repeated and averaged 100 times before it is transferred to computer through a GPIB (IEEE-488) databus for further analysis.

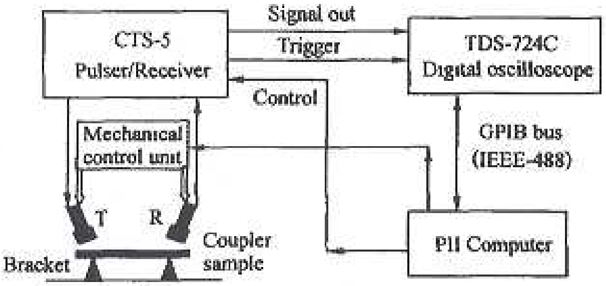


Fig. 3 Schematic of the experimental set-up

The measured material is a kind of the unidirectional glass fiber enhanced polyethersulfone (PES), the parameters of which are provided by the vendor in Pennsylvania. We calibrate the thickness by electrical micrometer, the measure error is ±2.54 μm. The density is measured by using of the Archimedes principle. The nominal values of the thickness and density are listed in the table of the experimental results below. The whole experimental procedure is under the temperature of $23 \pm 0.1°$.

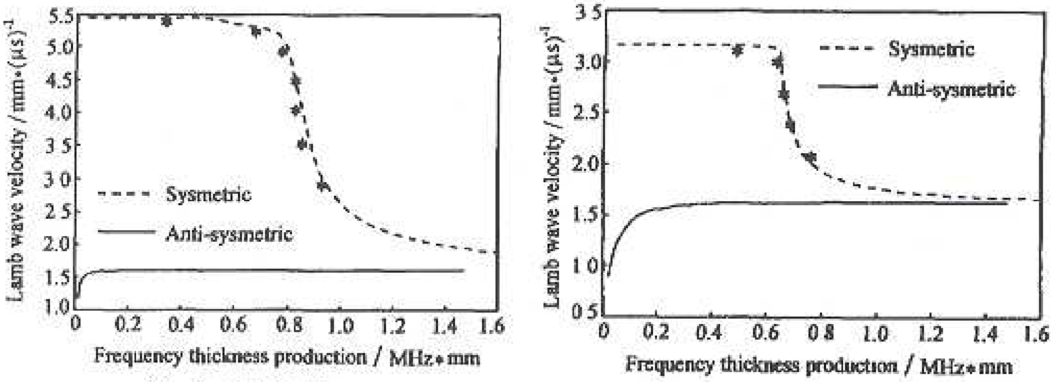
3.2 Frequency analysis method for leaky lamb wave

The Fourier analysis method, instead of the conventional tone burst swept frequency technique, is used to analyze the received leaky Lamb wave signals in order to construct the dispersion curves in the elastic layers. The received leaky Lamb wave signals are Fourier transformed to obtain a magnitude versus frequency spectrum, and the peaks in the spectrum indicate the presence of a Lamb wave root. The phase velocities for the Lamb waves are determined by θ, adjusted by a Mechanical Control Unit. The Lamb wave phase velocity can be calculated by Snell's law,

$$v_{\text{Lamb}} = \frac{v_L}{\sin(\theta)}, \tag{18}$$

where v_L denotes the ultrasound velocity in water and is equal to 1.49 mm/μs since all experiments are carried out under a constant temperature of 23°C. The Lamb roots are measured for a set of angles θ_j, i.e., for a set of phase velocities. The transducer holder mechanical structure controls the angle in the range of 10° to 54° with the step of 15′, which corresponds to the phase velocities of the Lamb waves approximately in the range from 1900 m/s to 6000 m/s.

Denoted by the dot '$*$', the experimental dispersion data of the unidirectional glass fiber enhanced polyethersulfone (PES) with the thickness of 244 μm tested by our experimental system is shown in the Fig. 4.



(a) Z-axis direction

(b) Y-axis direction

Fig. 4 The experimental data of the Lamb wave dispersion curve

Material: The unidirectional glass fiber enhanced polyethersulfone (PES), $h = 244$ μm)

4 Results and discussions

The evaluation results for the material by low frequency symmetrical Lamb wave propagating in the direction of Z-axis and Y-axis are shown in the Tables 1 and 2.

Table 1 The parameter evaluation results by using the low frequency ultrasonic Lamb wave propagating in the direction of Z-axis

Material parameters	Bulk wave method	Characterizing method by low frequency ultrasonic Lamb wave propagating in the direction of z-axis	
h/mm	0.244	0.250	2.3%
$\rho/(\mathrm{g}\cdot\mathrm{cm}^3)$	1.650	1.691	2.4%
C_{11}/GPa	18.22	16.99	−6.7%
C_{13}/GPa	2.45	2.55	4.1%
C_{33}/GPa	50.34	49.23	−2.2%
C_{44}/GPa	4.54	5.58	0.9%

Table 2 The parameter evaluation results by using the low frequency ultrasonic Lamb wave propagating in the direction of Y-axis

Material parameters	Bulk wave method	Characterizing method by low frequency ultrasonic Lamb wave propagating in the direction of y-axis	
h/mm	0.244	0.247	1.1%
$\rho/(\mathrm{g}\cdot\mathrm{cm}^3)$	1.650	1.608	−2.5%
C_{11}/GPa	18.22	17.85	−2.0%
C_{12}/GPa	8.94	9.19	2.8%

It is shown from Tabs. 1 and 2 that we can characterize thickness and density of the ultra-thin unidirectional fiber-enhanced laminate composite by low frequency Lamb wave propagating in the direction of Z-axis and Y-axis with h down to 0.1λ, and the evaluation accuracy for C_{11} is better by using the Lamb wave propagating in the direction of Y-axis.

Moreover, we deduced that the evaluation errors of our methods are influenced by the possible measurement errors of the Lamb wave velocities, which are directly related with the angle resolution of the transducer holder. The possible measurement errors of the Lamb wave velocities at different incidence angles with the angle resolution of 15′ is shown in Fig. 5.

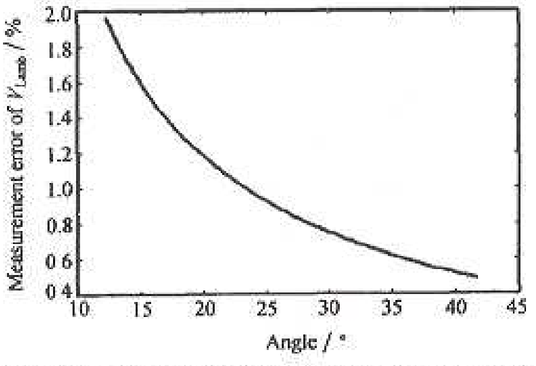


Fig. 5 The possible measurement errors of the Lamb wave velocities at different incidence angles with the angle resolution of 15′

It is clear that the possible measurement errors of the Lamb wave velocities in our experiment is about 2% since the angle range of our experiment around 16°. We also noticed that the estimation errors of the low frequency ultrasonic Lamb wave methods also depends on the beam shape and the frequency band of the transducer and dispersion characteristics of the ultrasonic Lamb wave.

5 Conclusions

By theoretical analysis and experimental testing, a low-frequency multi-mode ultrasonic Lamb wave method is proved to be suitable for characterizing thickness, density and elastic constants of the ultra-thin transversely isotropic laminate composite. We also draw the following conclusions:

(1) By using low frequency Lamb wave we can characterize the unidirectional glass fiber enhanced polyethersulfone with thickness down to ten percent of the ultrasonic wavelength.

(2) According to sensitivity analysis, the symmetrical Lamb wave propagating in the direction of Z-axis is proper to characterize ρ, d, C_{13}, C_{33} and C_{44}. On the other hand, ρ, d, C_{11} and C_{12} could be evaluated by the symmetrical Lamb wave propagating in the direction of Y-axis. The evaluation errors are less than tow percent for all parameters but C_{13} and C_{12}.

(3) The accuracy of our methods depends on the frequency band, beam characteristic, angle resolution of the transmitting and receiving transducers, and the dispersion property of the thin composite. The method is promising to characterize the acoustic and mechanical parameters of other kinds of thin laminate composites, which will be reported in the future.

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