Acoustic band-gap engineering using finite-size layered structures of multiple periodicity

Mingrong Shen[a)] and Wenwu Cao[b)]
Materials Research Laboratory, The Pennsylvania State University, University Park, Pennsylvania 16802

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The transmission coefficient of a layered structure made of glass and water was calculated using transfer matrix method and also measured as a function of frequency. It was found that acoustic band gaps can be created using only 3–4 cells of a two-phase layered structure. By introducing two or more periods into the layered structure, very sharp passbands and very broad stopbands can be engineered for acoustic waves. Such acoustic band-gap materials could be used for making high-quality acoustic filters, acoustic mirrors and vibration insulation devices in selective frequency range. © *1999 American Institute of Physics.* [S0003-6951(99)01549-1]

Photonic band-gap engineering in mesoscopic systems has been successfully realized by using crystals with periodic structures.[1,2] The fundamental principle is to use the periodic medium to regulate the propagation of electromagnetic waves. In the frequency window of the band gaps, electromagnetic waves cannot go through the crystal. In principle, the band-gap phenomena can be produced when the wavelength of the propagating waves approaches the period of the medium. As an analogue to photonic band-gap engineering, such band-gap phenomena also occur in the case of acoustic waves propagating in a periodic medium with the period comparable to the wavelength. Passbands and stopbands are produced in the frequency spectrum. In the frequency range of an acoustic (or phonotic) stopband, sound waves or ultrasonic vibrations cannot propagate. Such acoustic band-gap materials can have many practical applications, such as elastic/acoustic wave filters, ultrasonic silent blocks, acoustic mirrors, and ultrasonic array transducers. In the past, some interesting microscopic physical phenomena have been duplicated using ultrasonic waves with scale-up designs, such as the Anderson localization of sound and vibrations;[3–5] it is therefore natural to extend the photonic band-gap concept to acoustic waves. This letter reports an interesting study of using multiple periodicity and acoustic impedance difference to engineer large band gaps and/or narrow passbands using layered structure of finite size.

The materials under study are comprised of repeated cells made of two different materials with large acoustic impedance difference. The length scale of the period is the same as the wavelength of the acoustic waves. There has been growing interest in the band-gap materials in recent years. Several theoretical methods were developed in the past few years to study the band-gap formation, for example, the transfer matrix method,[6] plane wave method,[7,8] and effective medium method.[9] The first experimental study on ultrasonic band gaps reported in the literature was the measurement of sound attenuation in a sculpture.[10] Ultrasonic band gaps were also experimentally observed recently in one-dimensional arrays and two-dimensional periodic composites for longitudinal waves.[11,12]

In this letter, we report a combined theoretical and experimental study on a finite one-dimensional layered medium containing two different periods. The main objective is to use the idea of band-gap engineering to fabricate materials that can have narrow passbands and/or very broad stopbands.

As shown in Fig. 1, the structure is made of two substructures with period d_1 and d_2, respectively, and each of them consists of four cells. Each substructure can produce its own band structures. If the passbands of the two substructures are not overlapped in certain frequency range, acoustic waves cannot pass through the entire structure creating a broader stopband; if they are partially overlapped, only the overlapped frequencies can pass through, thus, a filter of narrow passband could be obtained.

In Fig. 1, element type 1 and type 3 are glass plates while type 2 and type 4 are both water. A longitudinal acoustic plane wave was sent from the left at $x=0$. At time t, the wave function in the nth layer can be written as

$$\psi_{nj}=A_{nj}e^{i(2\pi ft-k_jx)}+B_{nj}e^{i(2\pi ft+k_jx)}$$

$$(j=1-4,\ \ n=1-N), \tag{1}$$

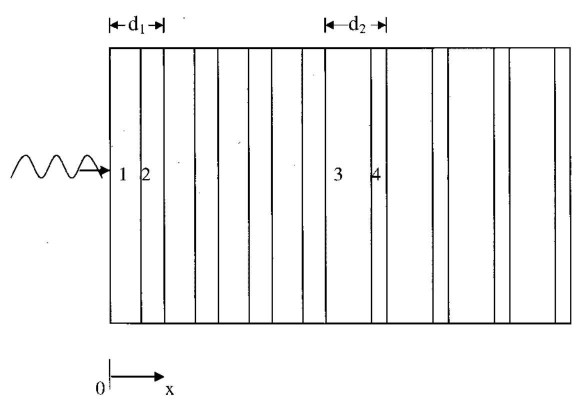

FIG. 1. Schematic of the layered structure studied, which consists of two periodic substructures with period d_1 and d_2, respectively. Materials 1 and 3 are two different glasses and materials 2 and 4 are both water.

[a)]Permanent address: Physics Department, Suzhou University, Suzhou 215006, People's Republic of China.
[b)]Electronic mail: cao@math.psu.edu

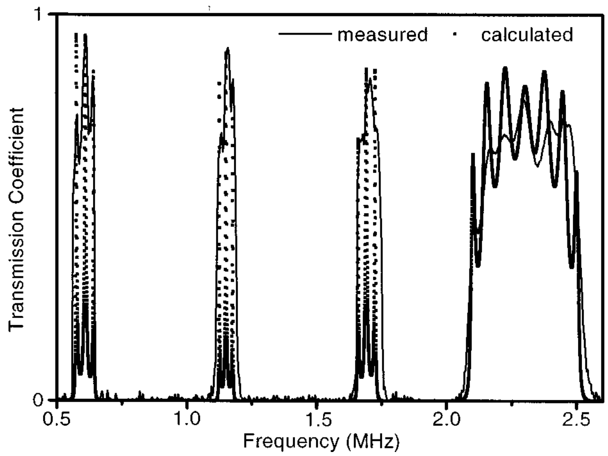

FIG. 2. Calculated and measured transmission coefficient as a function of frequency for a layered medium containing glass (1) and water (2). The thickness of glass is 1.23 mm and the water gap is 1.27 mm.

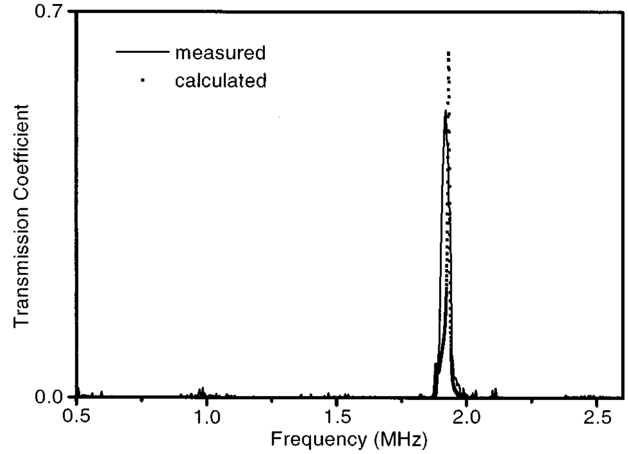

FIG. 3. Calculated and measured transmission coefficient for the structure shown in Fig. 1 as a function of frequency. The thickness of materials 1, 2, 3, and 4 are 1.23, 1.14, 3.00, and 0.51 mm, respectively.

where the first and second terms on the right-hand side of Eq. (1) represent the forward and reflected waves, respectively, $k_j = 2\pi f / c_j$ is the wave vector, and f and c_j are the frequency and phase velocity, respectively. The indices n and j are the cell number and material type, respectively.

The transfer matrix technique of Ref. 6 was used to calculate the acoustic band structures for the design given in Fig. 1. The properties of the materials used in the calculations are as follows:

Material 1: glass with density 2459 kg/m^3 and phase velocity 5660 m/s.

Material 3: glass with density 2767 kg/m^3 and phase velocity 5784 m/s.

Materials 2 and 4: distilled water with density 1000 kg/m^3 and phase velocity 1480 m/s.

Dissipation was included in the numerical calculations by adding a small imaginary component to the phase velocity in the glasses, i.e., $\tilde{c}_1 = (5660 + 10i)$ m/s and $\tilde{c}_3 = (5784 + 10i)$ m/s.

The experiments were conducted in a water tank using a setup similar to the one described in Refs. 12 and 13. Two broadband ultrasonic transducers were used for the measurements, one as transmitter and the other as receiver. In order to cover the interested frequency range of 0.5–2.6 MHz, two pairs of broadband ultrasonic transducers were used. The center frequencies of the transducers are 2.5 and 1.5 MHz, respectively, and the nominal active diameter of the transducers is 12 mm. The transmitting transducer was driven by a DPR35 pulser/receiver and the transmitted signal was received by a digital oscilloscope (Tektronix TDS 460A with fast Fourier transform analysis capabilities), then downloaded to a personal computer. Ten-signal average scheme was used to improve the signal-to-noise ratio during the measurements.

Shown in Fig. 2 are the calculated and measured frequency spectra of the transmission coefficient for a periodic layered structure with dimensions given in the figure caption. There are four passbands between the frequencies of 0.5 and 2.6 MHz. Gibbs type oscillations can be clearly seen in the calculated passbands and they are also confirmed by experiments within the experimental error. The Gibbs type oscillations here is similar to that reported in Ref. 12.

Figure 3 shows the calculated and measured frequency spectra of the transmission coefficient for the structure shown in Fig. 1, which has two different periods. It is intriguing to see that a very narrow passband is indeed obtained near 1.9 MHz. The agreement between the theoretical and experimental results is very good. Figure 4 shows the frequency spectra of the transmission coefficient of another design, for which the two substructures do not have overlapped passbands. As a result, no significant signal can pass through in the whole measured frequency range. Again, the calculated and experimental measured results agreed very well.

In conclusion, we have successfully engineered acoustic band-gap materials with either a very broad stopband or a very sharp passband. By changing the dimensions and type of materials used in the structure, the acoustic band-gap engineering concept of using multiple periodicity can be generalized to higher or lower frequency ranges. The produced acoustic band-gap materials have many potential applications, such as acoustic filters allowing only selective frequencies to pass through, ultrasonic silent block which can provide a vibration-free environment for high precision devices, and acoustic mirrors.

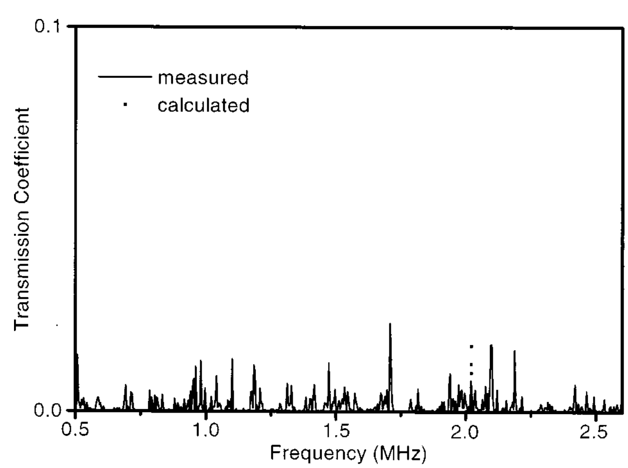

FIG. 4. Calculated and measured transmission coefficient as a function of frequency for another design of the structure shown in Fig. 1. The thickness of materials 1, 2, 3, and 4 are 1.23, 1.27, 3.00, and 0.51 mm, respectively.

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