Special Issue Correspondence

Finite Element Analysis of Periodic and Random 2-2 Piezocomposite Transducers with Finite Dimensions

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Abstract—A finite size periodic 2-2 composite transducer has been studied using finite element method, and the numerical results are verified by experiments. The focus of this study is on the mode coupling between the thickness resonance and pitch resonance. Such coupling effectively limits the pitch size of the composites preventing them being used in high frequencies. The effects of randomness is also being investigated using finite element analysis. The pitch resonance can be effectively suppressed by introducing 50 to 80% randomness in the structure. Numerical results also show that the decoupling of the thickness resonance and pitch resonance can increase the electromechanical coupling coefficient k_t for the thickness mode and shifts the thickness resonance frequency.

I. Introduction

Piezocomposite materials have been widely used in making transducers for medical ultrasonic imaging and underwater acoustics owing to their superior properties over ceramics or polymer single phase materials [1]–[4]. However, the multiphase structure of a piezocomposite transducer makes it much more difficult to analyze compared to a single phase transducer. One must rely on numerical methods to model such structures [5]–[13].

A typical 2-2 composite transducer is illustrated in Fig. 1; it consists of periodic ceramic and polymer layers in X_1-direction. The operation direction, which is also the ceramic poling direction, is along X_3. Material properties for the composite analyzed in this paper are given in Table I. Due to the periodic nature of 2-2 composite transducers, there are lateral modes associated with the pitch size [13], [14]. These lateral modes can couple to the thickness mode affecting transducer operation, particularly at high frequencies. The two main consequences of the mode coupling are the reduction of the thickness electromechanical coupling coefficient and the prolonging of pulse ringdown.

In this paper, we report a study on a finite 2-2 composite system using finite element method (FEM). The main focus will be on the lateral mode frequencies, the associated vibration profiles for the resonances, and the effect of randomization. For this purpose, the transducer is designed to have the pitch resonance very close to the thickness resonance. This situation could be found in high frequency array structures because the element size cannot be too small. Therefore, this study on the 2-2 composite structure can help a great deal on the design of high frequency linear and phased arrays used in medical ultrasound.

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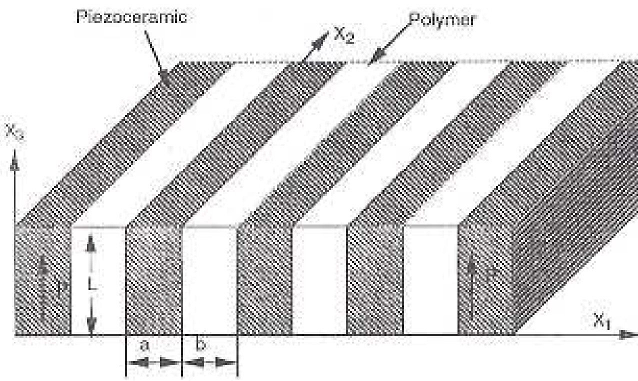

Fig. 1. Schematic of a 2-2 ceramic-polymer composite.

II. Finite Element Analysis

Finite element analysis (FEA) is the most powerful tool available today in designing composite transducers. It can handle complex structures with irregular boundaries. The FEA package used for the present study is ANSYS®.

A. Modal Analysis and Harmonic Analysis

Modal analysis calculates the resonance frequencies and the corresponding deformation shape of the structure. The vibration profile corresponding to the resonance can give us a clear picture of the relative deformation and help us to identify the modes. For simplicity, a 2-D model is constructed with the left side boundary being symmetric and the right side boundary free. The deformation profile of the first lateral mode and the thickness mode for this composite are shown in Figs. 2(b) and (c), respectively. The resonance frequencies calculated using ANSYS® agree with experimental results [17]. In the thickness mode both ceramic and polymer are vibrating in phase, while in the first lateral mode, they are 180° out of phase. Because the system is finite, strong edge effects are shown on the right-hand side of the model where free boundary condition was imposed.

Harmonic analysis in ANSYS® is designed to study the steady state response of the structure under an AC electric field at a specified frequency. From the electric charge output at the transducer surface, one can calculate the electrical admittance of the piezocomposite. Both the resonance and antiresonance frequencies can be found on the admittance curve as shown in Fig. 3, in which the first peak corresponds to the thickness mode and the second peak represents the first pitch resonance.

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TABLE I
MATERIAL PROPERTIES FOR PZT 5H AND POLYMER.

PZT 5H properties						
Piezoelectric coefficients, dielectric constants, density and Rayleigh stiffness damping (β)						
e_{31} (C/m^2)	e_{33} (C/m^2)	e_{15} (C/m^2)	ε_1 (10^{-8} C/m)	ε_3 (10^{-8} C/m)	ρ (kg/m^3)	β (10^{-10} s)
−6.3	23.5	17.1	1.5	1.3	7500	3.0
Elastic constants c^E (10^{10} N/m^2)						
c_{11}	c_{12}	c_{13}	c_{33}	c_{44}	c_{66}	
13.0	8.3	8.8	12.1	2.3	2.3	
Polymer properties						
Density, Young's modulus, shear modulus, dielectric constant and Rayleigh stiffness damping						
ρ (kg/m^3)	E (10^9 N/m^2)	G (10^9 N/m^2)	ε (10^{-11} C/m)	β (10^{-10} s)		
1097	3.5	1.3	3.44	70.0		

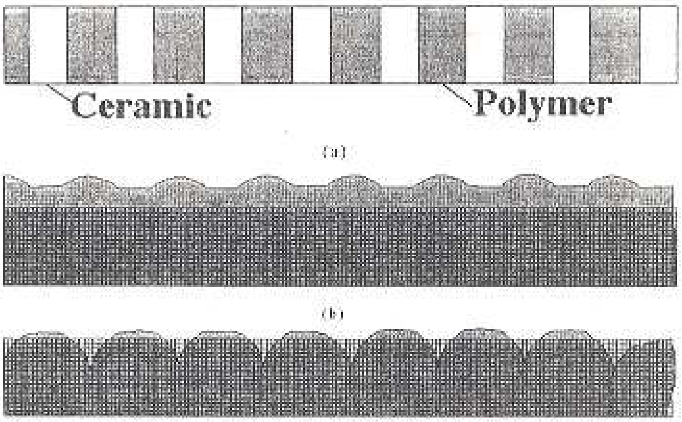

Fig. 2. Calculated vibration profiles: (a) The FEA model; (b) Thickness resonance; (c) First lateral mode.

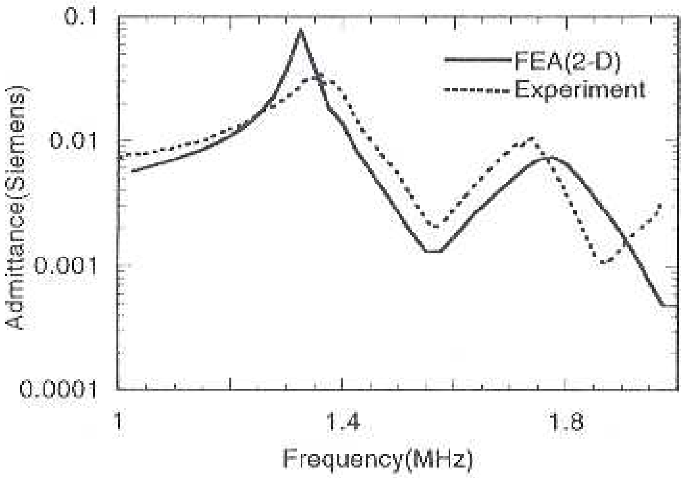

Fig. 3. Calculated and measured admittance versus frequency for a periodic 2-2 composite.

B. Lateral Modes in a 2-2 Composite

The lateral modes are not desired in transducers because they produce interference to the thickness mode. How to eliminate these lateral modes is one of the most difficult challenges in designing high frequency composite transducers. Conventional transducer design will limit the operating frequency of the composite transducer to be half of the pitch resonance in order to avoid mode coupling effect [18]. For high frequency transducers (> 20 MHz), such fine pitch is very difficult to manufacture in reality. The question is if one can design high frequency composite transducers without having to make ultrafine pitch.

In order to analyze the mode coupling behavior, we have designed a transducer with the thickness resonance very close to the first lateral pitch resonance. The thickness resonance frequency for this model composite transducer is 1.26 MHz, and the first lateral resonance frequency is 1.74 MHz. The resonance patterns shown in Fig. 2 were obtained by doing modal analysis using ANSYS®. Figs. 2(b) and (c) show the vibration profiles of the thickness and the lateral modes, respectively. One can see that the surface vibration of a 2-2 composite is quite nonuniform, even when the polymer and the ceramic are vibrating in phase in the thickness mode.

In this model design the thickness and the first lateral mode are strongly coupled. This coupling is reflected in the admittance curve shown in Fig. 3 (solid line) in terms of the peaks overlapping. The modal shapes for the resonance and antiresonance can be obtained from modal analysis under short circuit and open circuit conditions.

In order to verify the FEA results, a 2-2 composite transducer of the same design was constructed, and the admittance curve was measured by using a HP4194A impedance analyzer (dashed line in Fig. 3). The resonance frequencies obtained from the experiment are 1.34 MHz and 1.74 MHz, respectively, for the thickness and pitch resonance, which match well with the prediction from the FEA (solid line in Fig. 3). The absolute values of the calculated admittance show some deviation from the experiments due to the fact that the damping factor is not exact in the FEA. The Rayleigh damping constants α and β are used to define the damping effect in the numerical analysis. The α damping (or mass damping) is ignored ($\alpha = 0$) and the β damping (or stiffness damping) is taken as a constant over a frequency range; therefore, the damping ratio $\xi = \beta\omega/2$ is assumed to be proportional to frequency in a narrow frequency range. This may not reflect the experimental situation. Therefore, the use of Rayleigh damping may introduce certain errors into the FEA results. Another way of defining damping in ANSYS® is to define the damping ratio ξ as a constant; however, one can only define one global damping ratio in ANSYS®, error will be introduced if there are more than one types of materials, such as the composite situation. The damping parameter for the ceramic is an order of magnitude smaller than that for the polymer, therefore, the polymer damping parameter dominates the damping effect. Fig. 4(a) shows the calculated admittance curves for two different input damping parameters

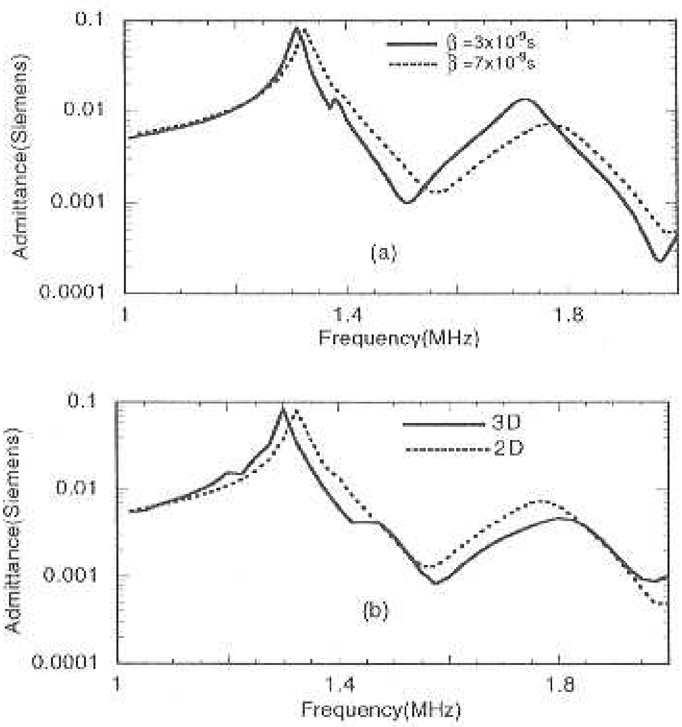

Fig. 4. (a) Calculated admittance curves of a 2-2 composite with different Rayleigh stiffness damping in the polymer: $\beta = 3\text{e-}9$ s (solid line) and $\beta = 7\text{e-}9$ s (dashed line). For both cases, the Rayleigh stiffness damping of ceramic is set to be $\beta = 3\text{e-}10$ s. (b) The calculated admittance curves using 2-D and 3-D FEA models.

for the polymer. As we can see in Fig. 4(a), the thickness resonance peak does not change much with the change of polymer damping, but the lateral resonance peak was suppressed. This suggests that the lateral modes can be reduced by filling lossy polymer in composite transducer, a general practice in the transducer industry. The inaccurate input material properties is another error source in the FEA. Generally speaking, the manufacturer-supplied values contain 10 to 20% fluctuations.

As a comparison to the 2-D model, 3-D model was also constructed using ANSYS®. For the structure shown in Fig. 1, 64 MBs of RAM and 1GBs of hard disk were required to perform the harmonic analysis in the frequency range from 1 to 2 MHz with 10 kHz increment. The mesh size has strong effect on the accuracy of the FEA results in 3-D. As a rule of thumb, the mesh size should be at least one-eighth of the smallest wavelength from our experience. Slight difference between the 2-D and 3-D models was found as shown in Fig. 4(b). First, there are more bumps in the 3-D admittance curve caused by the finite dimension in the X_2 direction; second, the peak positions are slightly shifted. The difference reveals that the X_2 dimension of a 2-2 composite must be much larger than the other two dimensions in order for 2-D model approximation to be valid.

The 2-2 composite being studied using FEA has 17 cells, with $b = 0.362$ mm (kerf) and $a = 0.273$ mm (element). The thickness along X_3 is $L = 1.12$ mm. The dimension in X_2 direction is 12 mm which is sufficient to justify using a 2-D FEA model. Such a design makes the thickness and pitch resonances couple strongly so that we can study the coupling effect. Conventional method to decouple the two modes is to make finer pitch, which is hard to achieve for high frequency transducers when the pitch size needs to be smaller than 30 μm. Another possibility to suppress the pitch resonance is to introduce randomness into the structure as predicted from the extended T-matrix calculation [16]. This case is evaluated here further using FEA.

C. Random 2-2 Composite Transducers

Our previous study using extended T-matrix technique showed that the first lateral mode can be suppressed by introducing randomness into the structure [15], [16]. In order to verify the results and to visualize the effects, an FEA model is conducted. As pointed out earlier [15], [16], it is more effective to randomize the kerf width when the ceramic content is low, and it is more effective to randomize the ceramic size when the ceramic content is high (> 60%). The ceramic volume percentage of the 2-2 composite studied in our FEA is 44%; therefore, randomization is done in the polymer phase. During randomization, the volume percentage of ceramic is maintained at the same level as the regular composite studied in previous section.

The random kerf width is chosen according to the following formula:

$$b_i = (1-c)b_o + c\frac{Nr_i}{\sum_{i=1}^{N} r_i} b_o \tag{1}$$

where b_o is the arithmetic mean of the polymer width, N is the total number of cells in the composite, r_i is a set of random numbers between 0 and 1, and c is a value between 0 and 1 representing the fraction of randomization.

As shown in Fig. 5, when the randomness c is 20%, the admittance peak corresponding to the lateral mode begins to split. At 50% randomness, the peak value of the lateral mode is greatly reduced, and the lateral mode is practically suppressed when the randomness is increased to 80%. One may also find that the thickness frequency has been shifted up after the lateral mode being suppressed. The thickness frequency is 1.25 MHz, and 1.296 MHz with and without the coupling as shown in Figs. 5(a) and (e), respectively. This may be understood as: the coupling between the thickness mode and the lateral mode tends to push down the thickness resonance frequency and push up the lateral resonance frequency shown in Fig. 3. When randomness destroys the lateral mode, the thickness resonance frequency will be shifted upward to go back to its original position. This random effect agrees quite well with the calculations using extended T-matrix [15], [16].

Because the antiresonance frequency has a much larger shift from 1.5 MHz to 1.74 MHz, the electromechanical coupling coefficient k_t for the thickness mode is increased through the introduction of randomness. The thickness coupling coefficient in Fig. 5(a) is 0.6; in Fig. 5(e) it is 0.7. In other words, the energy loss to the lateral mode is recovered.

III. Summary and Conclusions

We have performed FEA and experimental investigations on the thickness resonance of a 2-2 composite transducer. The resonance frequencies can be obtained by doing either modal analysis or harmonic analysis using ANSYS®. The surface displacement of a 2-2 composite is quite nonuniform in all the modes. Very importantly, we found that only in the decoupled thickness mode the ceramic and polymer are vibrating in

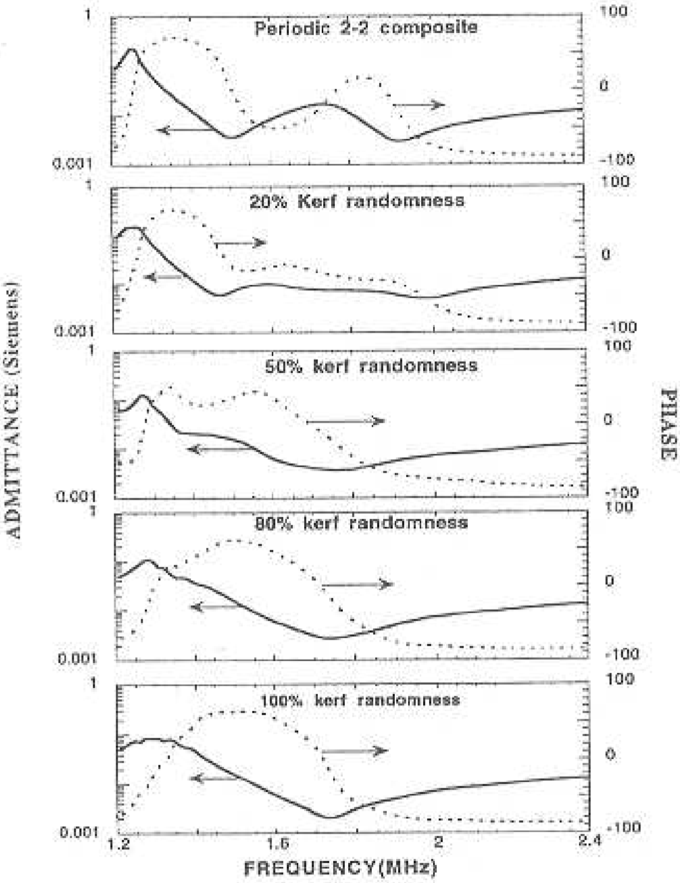

Fig. 5. Admittance calculated by FEA for a 2-2 composite with randomized kerf width. The pitch resonance is gradually destroyed with the increase of randomization, and the resonance and antiresonance frequencies for the thickness mode are shifted upward.

phase; in other cases, such as in the pitch resonance, they are out of phase. FEA has also been used to analyze a finite size 2-2 composite with random kerf width. From the calculated admittance curves, we have demonstrated the gradual reduction of the first lateral mode with the increase of randomness in the polymer width. It is concluded that, with some degree of damping, 50% randomness is sufficient to suppress the first lateral mode. The FEA results agree very well with the analysis of extended T-matrix on random 2-2 composite [15], [16]. The results obtained here can be used to help the design of 1-D linear and phased arrays, which have the same structure but different driving mechanism.

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