

Finite Element Analysis and Experimental Studies on the Thickness Resonance of Piezocomposite Transducers

Wenkang Qi* and Wenwu Cao*†

**Intercollege Materials Research Laboratory, †Department of Mathematics, The Pennsylvania State University, University Park, Pennsylvania 16802*

Finite element method (FEA) has been used to calculate the thickness resonance frequency and electromechanical coupling coefficient k_t for 2-2 piezocomposite transducers. The results are compared with that of the effective medium theory and also verified by experiments. It is shown that the predicted resonance frequencies from the effective medium theory and the unit cell modeling using FEA deviate from the experimental observations for composite systems with a ceramic aspect ratio (width/length) more than 0.4. For such systems, full size FEA modeling is required which can provide accurate predictions of the resonance frequency and thickness coupling constant k_t .

KEY WORDS: Aspect ratio; composite transducers; effective medium theory; finite element analysis; piezocomposites. © 1996 Academic Press, Inc.

1. INTRODUCTION

A transducer is usually characterized by two major properties: sensitivity and resolution. The sensitivity is related to the electromechanical coupling coefficient, while the resolution is related to the center frequency and bandwidth. At the beginning of the ultrasonic imaging industry, two types of piezoelectric materials were used as transducer materials: lead zirconate titanate (PZT) and polyvinylidene fluoride (PVDF). PZT has high acoustic impedance, making it very difficult to send ultrasonic energy into the human tissue, which has very low acoustic impedance. In addition, the Q value of PZT is very high so that the bandwidth is narrow resulting in poor resolution due to ringing effects. On the other hand, PVDF has a very good acoustic impedance match with human tissue, but its electromechanical coupling coefficient is very low, resulting in low sensitivity. In addition, the low dielectric constant of PVDF also creates the problem of electric impedance mismatch, which limits the application of PVDF in array transducers (Table I).

The advent of piezoelectric composites greatly improved this situation [1,2]. Piezocomposites have large coupling coefficients as well as low acoustic impedance, making them ideal transducer materials. Nowadays, piezoelectric composites are widely used in making underwater acoustic and medical ultrasonic transducers [2-4]. However, due to the biphasic nature and the large difference in the elastic stiffness between the polymer and the ceramic, the surface displacement is often nonuniform [5-8]. It is therefore difficult to accurately predict the resonance frequency of the composite transducers using simplified models.

The most commonly-used method for designing composite transducers is the effective medium model [4]. Experience reveals that the actual resonance frequency of the designed

TABLE I
Material Properties of PZT and Epoxy

	Elastic compliance (10^{-12} m ² /N)									
	s_{11}^E	s_{33}^E	s_{12}^E	s_{13}^E	s_{44}^E	s_{66}^E				
PZT	16.5	20.7	-4.78	-8.45	43.5	42.6				
Epoxy	286.7	286.7	-97.9	-97.9	769	769				
Piezoelectric constants, d_{ij} , (10^{-12} C/N) dielectrics constants, k_{ij} , (ϵ_0) coupling constants, $k_{i\lambda}$ and k_t , and density ρ (kg/m ³)										
	d_{15}	d_{31}	d_{33}	$\epsilon_{11}^s/\epsilon_0$	$\epsilon_{33}^s/\epsilon_0$	k_{15}	k_{31}	k_{33}	k_t	ρ
PZT	741	-274	593	1700	1470	0.675	0.39	0.75	0.50	7800
Epoxy				4.0	4.0					1097

composite transducer is often lower than the theoretical estimates from the effective medium theory. Motivated by this discrepancy, we have conducted a combined experimental and finite element analysis to give a detailed assessment of the effective medium model and to derive the conditions for the application of such theoretical estimates. We also intended to evaluate the validity of the commonly used unit cell FEA modeling [9–11]. For simplicity, we only analyze a 2-2 composite transducer, but the conclusions are also valid for 1-3 type composite transducers.

2. EFFECTIVE MEDIUM MODEL FOR 2-2 PIEZOCOMPOSITES

A typical 2-2 composite is shown in figure 1. It is a layered structure of alternating polymer and piezoceramic constituents.

The constitutive relations for the polymer phase can be written as the following,

$$T_1 = c_{11}S_1 + c_{12}S_2 + c_{12}S_3 \quad (1a)$$

$$T_2 = c_{12}S_1 + c_{11}S_2 + c_{12}S_3 \quad (1b)$$

$$T_3 = c_{12}S_1 + c_{12}S_2 + c_{11}S_3 \quad (1c)$$

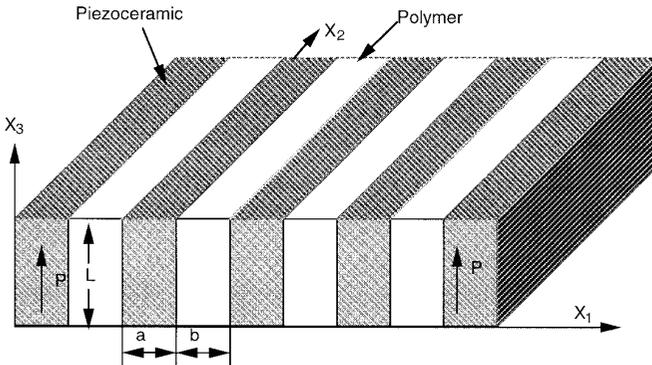


FIG. 1. Configuration of 2-2 composite investigated in this study.

$$T_4 = c_{44}S_4 \quad (1d)$$

$$T_5 = c_{44}S_5 \quad (1e)$$

$$T_6 = c_{44}S_6 \quad (1f)$$

$$D_1 = \epsilon_{11}E_1 \quad (1g)$$

$$D_2 = \epsilon_{11}E_2 \quad (1h)$$

$$D_3 = \epsilon_{11}E_3 \quad (1i)$$

Here, T_l and S_l ($l = 1, 2, \dots, 6$) are the stress and strain components, respectively, in Voigt notation [12], E_i and D_i ($i = 1, 2, 3$) are the electric field and electric displacement respectively, c_{ij} are the elastic stiffness constants and ϵ_{ij} are the dielectric constants.

Similarly, if we take the x_3 -direction as the poling direction, the constitutive relations in the ceramic phase can be written as:

$$T_1 = c_{11}^E S_1 + c_{12}^E S_2 + c_{13}^E S_3 - e_{31} E_3 \quad (2a)$$

$$T_2 = c_{12}^E S_1 + c_{11}^E S_2 + c_{13}^E S_3 - e_{31} E_3 \quad (2b)$$

$$T_3 = c_{13}^E S_1 + c_{13}^E S_2 + c_{33}^E S_3 - e_{33} E_3 \quad (2c)$$

$$T_4 = c_{44}^E S_4 - e_{15} E_2 \quad (2d)$$

$$T_5 = c_{44}^E S_5 - e_{15} E_1 \quad (2e)$$

$$T_6 = c_{66}^E S_6 \quad (2f)$$

$$D_1 = e_{15} S_5 + \epsilon_{11}^S E_1 \quad (2g)$$

$$D_2 = e_{15} S_4 + \epsilon_{11}^S E_2 \quad (2h)$$

$$D_3 = e_{31} S_1 + e_{31} S_2 + e_{33} S_3 + \epsilon_{33}^S E_3 \quad (2i)$$

where e_{il} are the piezoelectric constants, and the superscripts, E and S, refer to quantities at constant electric field and strain, respectively.

We can follow the same procedure as in [4] and use all the assumptions proposed there to derive the effective properties (denoted with an overbar) of a 2-2 composite,

$$\overline{c_{33}^E} = V \left[c_{33}^E - V' \frac{(c_{12} - c_{13}^E)^2}{V' c_{11}^E + V c_{11}} \right] + V' c_{11} \quad (3a)$$

$$\overline{e_{33}} = V \left[e_{33} - V' e_{31} \frac{(c_{13}^E - c_{12})}{V' c_{11}^E + V c_{11}} \right] \quad (3b)$$

$$\overline{\epsilon_{33}^S} = V \left[\epsilon_{33}^E + \frac{e_{31}^2 V'}{V' c_{11}^E + V c_{11}} \right] \quad (3c)$$

$$\overline{c_{33}^D} = \overline{c_{33}^E} + \frac{\overline{e_{33}^2}}{\overline{\epsilon_{33}^S}} \quad (3d)$$

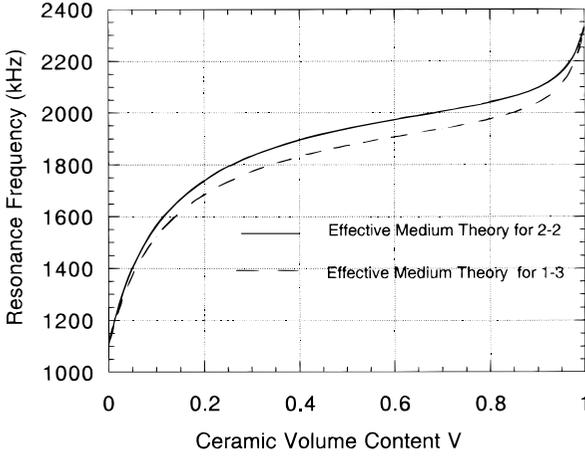


FIG. 2. Resonance frequencies for 2-2 and 1-3 composites calculated using the effective medium theory for different ceramic volume content.

$$\overline{h}_{33} = \frac{\overline{e}_{33}}{\overline{\epsilon}_{33}^S} \quad (3e)$$

$$\overline{\beta}_{33}^S = \frac{1}{\overline{\epsilon}_{33}^S} \quad (3f)$$

$$\overline{\rho} = V\rho^C + V'\rho^P \quad (3g)$$

In the above expressions, V and V' are the volume percentages of ceramic and polymer, respectively, $V' = 1 - V$, and ρ^P and ρ^C are the densities of the ceramic and polymer. Using the conventional definition, one can derive all the relevant effective quantities for the thickness mode operation,

$$\overline{k}_t = \frac{\overline{h}_{33}}{\sqrt{c_{33}^D \overline{\beta}_{33}^S}} = \frac{\overline{e}_{33}}{c_{33}^D \overline{\epsilon}_{33}^S} \quad (4)$$

$$\overline{Z} = \sqrt{c_{33}^D \overline{\rho}} \quad (5)$$

$$\overline{v}_l = \sqrt{\frac{c_{33}^D}{\overline{\rho}}} \quad (6)$$

$$f_r = \frac{\overline{v}_l}{2L} \quad (7)$$

\overline{v}_l and L are the longitudinal wave speed and the thickness of the composite in the poling (x_3) direction and f_r is the resonance frequency given by the effective medium theory.

Using the above equations, we have calculated the effective thickness resonance frequency for a 2-2 composite of 1 mm thick. Compared with a 1-3 composite transducer of the same thickness, the resonance frequency of a 2-2 composite is higher than that of a 1-3 composite of the same ceramic volume content (Fig. 2). The same is true also for the thickness coupling constant \overline{k}_t (Fig. 3).

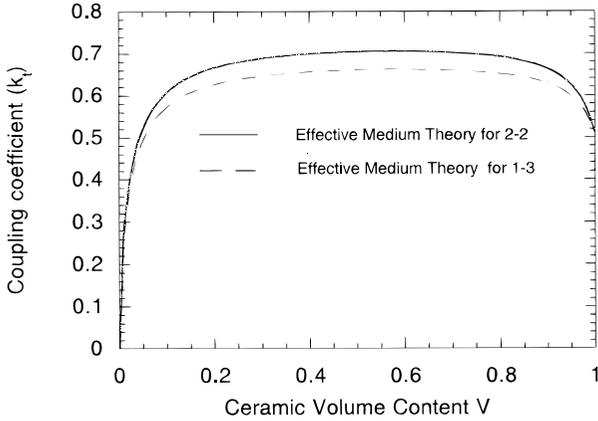


FIG. 3. Coupling constant k_t calculated for 2-2 and 1-3 composites of different ceramic volume content.

Although the effective medium theory is relatively simple and sometimes gives reasonable estimate for the resonance frequency, it fails to account for the aspect ratio effect, which can be substantial if the a/L (width/length) ratio is not sufficiently small [6,7]. For systems with large a/L ratio and low ceramic content, the isostrain assumption is no longer valid.

In addition, since a real transducer always contains a finite number of cells, one would not expect a very good match with experimental results from a unit cell model that automatically assuming periodic boundary conditions. For this reason, we have performed FEA on 2-2 composite transducers using both the unit cell and full dimension models.

3. FINITE ELEMENT ANALYSIS

The nonuniform displacement at the surface of composite transducers has been observed experimentally [5,8]. This inhomogeneity can greatly affect the overall performance of a transducer. For low frequencies, the situation may be treated by using elasticity theory and describing the two constituents separately. Some approximations can be used in solving the low frequency problem since there are no significant phase differences in the structure [6–8]. However, when the operating frequency is high and close to the thickness resonance, we must use FEA for an accurate theoretical prediction.

A commercial package ANSYS was used in our study and two models were analyzed: (1) A unit cell model, which was also analyzed by several other researchers [9–11]; (2) A finite real dimensional system. These FEA results are checked against our experiment results.

After some test runs, we found that the results from a 2-D model are almost the same as those from a 3-D model for the geometry we have chosen. Therefore, for computational efficiency, we performed only 2-D modeling. The models and the coordinate system are shown in figure 4.

One of our objectives is to study the change of the thickness resonance frequency and the electromechanical coupling coefficient k_t with respect to the change of ceramic aspect

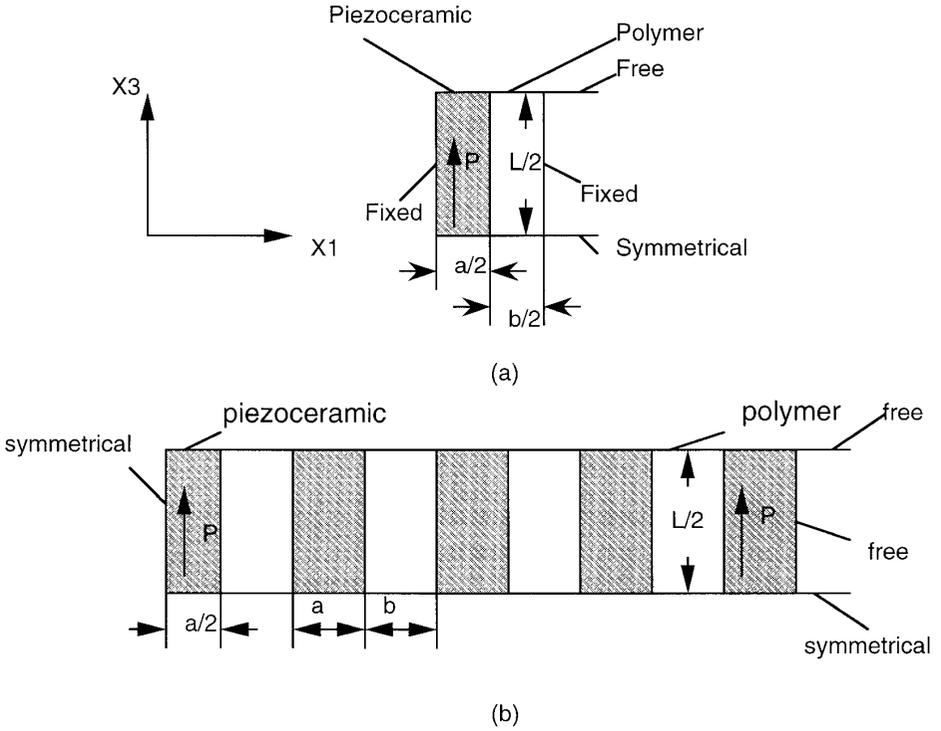


FIG. 4. 2-D models used in the finite element analysis. (a) unit cell model; (b) full dimension model.

ratio. Both the thickness resonance and the anti-resonance frequencies were calculated. The resonance frequency is calculated under short circuit condition (constant E) while the anti-resonance frequency is calculated in open circuit condition (constant D) [13]. From these two resonance frequencies, the electromechanical coupling coefficient k_t can be calculated using the formula,

$$k_t^2 = \frac{\pi f_r}{2f_a} \tan\left(\frac{\pi(f_a - f_r)}{2f_a}\right) \quad (8)$$

where f_r and f_a are the resonance and anti-resonance frequencies, respectively.

First, we performed analysis on the unit cell model. Only a quarter of the unit cell is needed due to symmetry (Fig. 4a). A composite of real dimensions was then analyzed. Again, only a quarter of the piece was analyzed due to symmetry (Fig. 4b). The results are plotted in figure 5 together with the experiment results.

4. RESULTS AND DISCUSSIONS

In order to verify the theoretical results, we made a series of 2-2 composite transducers using PZT-5H and Spurr's epoxy. The dimension along the x_2 -axis (into the paper) is made large enough so that the system can be treated as two dimensional. We start by making

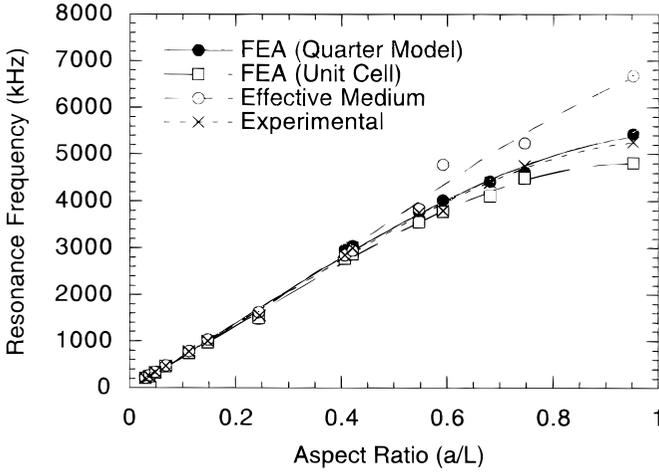


FIG. 5. Comparison of observed resonance frequencies and theoretical predictions from the effective medium theory, unit cell and full dimension finite element models at different ceramic aspect ratios. The widths of the ceramic and polymer are $a = 0.273$ mm and $b = 0.362$ mm, respectively.

a thick 2-2 piezocomposite in the x_3 -dimension, and later gradually increased the a/L ratio by shortening L , i.e., shortening the x_3 -dimension without changing the other dimensions. After each cutting, the sample is re-electroded and the resonance frequency measured using a HP 4194A impedance analyzer. From the impedance curves, the resonance and anti-resonance frequencies can be obtained, and the electromechanical coupling coefficient k_t can be determined using Eq. (8).

Another experiment was also performed to check the dimensional effect in the x_1 -direction. In other words, reducing the number of cells in the composite structure to see if it affects the resonance frequency in the x_3 -dimension. Impedance measurements were also used as the means to characterize this effect.

Figure 5 shows the comparison of the resonance frequencies calculated by the effective medium theory, unit cell FEA and real dimensional model FEA together with the experimental results. When the ceramic ratio a/L is less than 0.4, all theoretical models agree quite well with the experimental observations. But for a/L greater than 0.4, the effective medium theory prediction is too high while the prediction from the unit cell FEA model is too low. Only the real dimensional model provide accurate prediction for the resonance frequency.

The coupling constant k_t calculated from effective medium theory is independent of the aspect ratio Fig. 3. However, experimental results demonstrate a fluctuation of k_t with change of aspect ratio Fig. 6. This fluctuation is mainly caused by the coupling between the thickness mode and other lateral modes or their higher harmonics.

When the thickness, L , is reduced, the resonance frequency is shifted to higher frequencies. Whenever the resonance frequency approaches one of the lateral modes or their higher harmonics, energy will be lost to the lateral modes and the coupling constant k_t is reduced. Further increase of the thickness resonance frequency may recover some of the lost energy through mode decoupling until reaching the next lateral mode, which causes another reduction of the coupling constant. Therefore, we expect the k_t value to

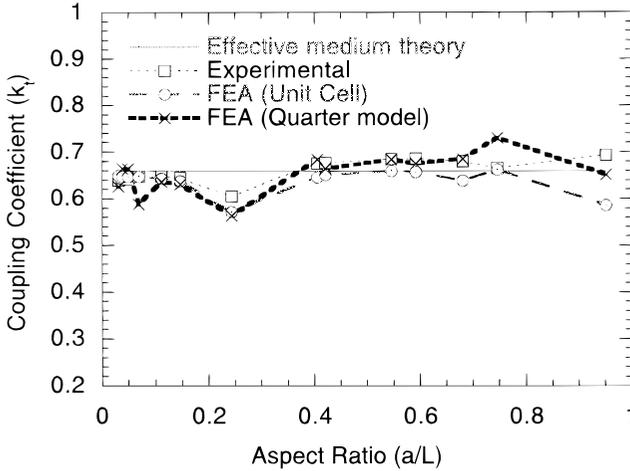


FIG. 6. Comparison of observed coupling constant k_t and theoretical calculations from the effective medium theory, unit cell and full dimension finite element models at different ceramic aspect ratios. The widths of the ceramic and polymer are $a = 0.273$ mm and $b = 0.362$ mm, respectively.

go up and down with the decrease in L . As shown in figure 6, this mode coupling effect is well accounted for by the FEA. Both the unit cell model and the real dimensional model show this fluctuating feature and the real dimensional model provides better agreement with the experimental observations.

The difference between unit cell and real dimensional model indicate that the resonance frequency in the x_3 -dimension will also depend on the composite size in the x_1 -dimension (number of cells). However, this effect is weak when the a/L ratio is small.

5. SUMMARY AND CONCLUSIONS

We have performed both experimental and FEA investigations on the resonance frequency of a 2-2 piezoelectric composite transducer and compared with the effective medium theory estimation. It is found that the effective medium theory gives good estimates when the a/L ratio is less than 0.4, but the calculation of the coupling constant is incorrect whenever the thickness mode gets close to one of the lateral modes. When the a/L ratio is larger than 0.4, the effective medium theory prediction will be higher than the actual resonance frequency of the composite transducer. The FEA results depend strongly on the details of the model. Unit cell modeling seems to underestimate the resonance frequency for composites with large a/L ratio but the fluctuation of k_t can be reasonably predicted. The most accurate modeling is the real dimensional FEA, which gave good predictions for both resonance frequency and coupling constant for all aspect ratios investigated.

ACKNOWLEDGMENTS

This work was supported by The Office of Naval Research under Grant No. N00014-93-1-0340 and The Whitaker Foundation under Special Opportunity Award.

REFERENCES

- [1] Newnham, R. E., Bowen, L. J., Klicker, K. A., and Cross, L. E., Composite piezoelectric transducer, *Materials in Engineering* 2, 93–106 (1980).
- [2] Smith, W. A., The Role of Piezocomposites in Ultrasonic Transducers, in *1989 IEEE Ultrasonic Symposium Proceedings*, pp. 755–766.
- [3] Hayward, G., and Hossack, J. A., Unidimensional Modeling of 1-3 Composite Transducer, in *1989 Ultrasonic International Conference Proceedings*, pp. 531–536.
- [4] Smith, W. A., Modeling 1-3 composite piezoelectric: thickness-mode oscillations, *IEEE Trans. Ultrason. Ferroelect. Freq. Contr.* 38, 40–47 (1991).
- [5] Auld, B. A., Kunkel, H. A., Shui, Y. A., and Wang, Y., Dynamic Behavior of Periodic Piezoelectric Composites, in *1983 IEEE Ultrasonics Symposium*, pp. 554–558.
- [6] Cao, W., Zhang, Q., and Cross, L. E., Theoretical study on the static performance of piezoelectric ceramic-polymer composites with 2-2 connectivity, *IEEE Trans. Ultrason. Ferroelec. Freq. Contr.* 40, 103–109 (1993).
- [7] Cao, W., Zhang, Q., and Cross, L. E., Theoretical study on the static performance of piezoelectric ceramic-polymer composites with 1-3 connectivity, *J. Appl. Phys.* 72, 5814–5821 (1992).
- [8] Zhang, Q. M., Cao, W., Zhao, J., and Cross, L. E., Piezoelectric performance of piezoceramic-polymer composites with 2-2 connectivity—a combined theoretical and experimental study, *IEEE Trans. Ultrason. Ferroelec. Freq. Contr.* 41, 556–564 (1994).
- [9] Hossack, J. A., and Hayward, G., Finite-element analysis of 1-3 composite transducers, *IEEE Trans. Ultrason. ferroelec. Freq. Contr.* 38, 618–629 (1991).
- [10] Hladky-Hennion, A., and Decarpigny, J., Finite element modeling of active periodic structures: application to 1-3 piezocomposites, *J. Acoust. Soc. Am.* 94, 621–635 (1993).
- [11] Hossack, J. A., Auld, B. A., and Batha, H. D., Techniques for Suppressing Spurious Resonant Modes in 1:3 Composite Transducers, in *1991 IEEE Ultrasonics Symposium*, pp. 651–655.
- [12] Wallace, W. C., Thermoelastic theory of stressed crystals and high-order elastic constants, *Solid State Phys.*: 25, 301–405 (1970).
- [13] Lerch, R., Simulation of piezoelectric devices by two- and three-dimensional finite elements, *IEEE Trans. Ultrason. Ferroelec. Freq. Contr.* 37, 233–246 (1990).