Influence of gas inertia and edge effect on squeeze film in near field acoustic levitation

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Detailed analysis showed that the existing theoretical models for near field acoustic levitation are not good enough to interpret the experimental data. The best existing models can only work in a small range of the squeeze numbers even with a fitting parameter. We report here a comprehensive nonlinear model, which takes into account the nonuniform surface vibration profile, gas inertia as well as entrance pressure drop at the film edge. Our model has no empirical parameters and the predicted results agree exceptionally well with our experimental measurements over a wide range of squeeze numbers. [doi:10.1063/1.3455896]

In microassembly lines, handling the components of microelectromechanical system presents a special challenge due to their fragile and surface-sensitive characteristics.[1] For such purposes, noncontact transport systems have many advantages, including wear free, no dust accumulation and without mechanical contact noises. Near field acoustic levitation (NFAL) method uses acoustic radiation to levitate an object in air from the radiating surface to a height much smaller compared to the wavelength. NFAL has been applied in industries for noncontact transportation of semiconductor wafers and other surface sensitive components.

There have been numerous experimental and theoretical studies on NFAL in the literature.[2–5] In most theoretical studies, the radiator was assumed to be rigid and perform pistonlike movement with uniform displacement across the vibrating surface. The gas film in between the vibrating surface and the object has been treated as a viscous fluid that satisfies the Reynolds equation. Experimental results showed that all existing theoretical models have some shortcomings. As detailed below that the simplest models lead to very large deviation from the experimental observation, while more advanced models only work in a small range of the squeeze numbers. Due to the fast development of nanomanufacture industries, there is an urgent need for more accurate theoretical models to help design better NFAL systems.

Recently, Liu *et al.*[6] studied experimentally the model dependent vibration shape of the radiator. Because the first and second resonance modes of the vibrator will produce different surface vibration profiles, one cannot use pistonlike motions to describe the experimental results. By using the experimentally measured surface profile as the radiating surface boundary condition for the squeeze film, the squeeze film model produced substantial improvement compared to pistonlike models.[6] However, the agreement between theory and experiments becomes worse as the squeeze number increases, which tells us that the deficiency of the squeeze film model is not limited to the uniform surface vibration assumption.

For thicker film and higher frequency oscillations, the Reynolds number increases and the gas inertia will play an important role in the acoustic region. Veijola[7] proposed a modified Reynolds equation using a relative flow rate coefficient Q_{pr} including gas inertia and rarefaction but the modification is only limited in gas damping and the linear model used by Veijola could not produce levitation force.

The edge effect of the squeeze film is important for the pressure boundary condition when gas inertia is included in the nonlinear model.[8] Turns[9] proposed a formula for the edge pressure drop in bearing films, which included an empirical entrance loss efficient Ce. This coefficient was later experimentally measured by Kuroda[10] to be around 2.0. Although this model works reasonably well for very large squeeze numbers, the theoretical results deviate from reality as the thickness of the squeeze film increases (squeeze number decreases) because the model assumed uniform radial velocity across the thickness and ignored the influence of squeeze film geometry.

In this paper, we present a comprehensive nonlinear model by taking into account the gas inertia, edge effect as well as the nonuniform surface vibration distribution. With more realistic physical consideration, we have derived a formula for the edge pressure drop, which depends on the radial velocity profile as well as the squeeze film aspect ratio α. Experimental validation indicates that our model can accurately predict the levitation force generated in NFAL for a wide range of squeeze numbers.

The problem under study involves a sound radiation plate of radius r_0 vibrating along the surface normal z-direction at a resonant frequency f. A reflector of the same radius is placed initially at a distance $h_0(<<r_0)$ from the vibrating surface. Considering the axisymmetric radial laminar air flow, the momentum equation for Newtonian fluids with inertia terms is given by

$$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+v_z\frac{\partial v_r}{\partial z}\right)=-\frac{\partial p}{\partial r}+\frac{\partial \tau_{rz}}{\partial z}, \tag{1}$$

where $\tau_{rz}=\mu(\partial v_r/\partial z)$ is the shear stress, p, ρ and μ are gas pressure, density, and viscosity, respectively; v_r is the velocity component along r.

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For thin layer air film, we assume that the pressure is a function of r only so that

$$\frac{\partial p}{\partial z} = 0. \tag{2}$$

The continuity equation is given by

$$\frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{\partial v_z}{\partial z} = 0, \tag{3}$$

where v_z is the velocity components along the z direction. The boundary conditions of the velocity components on the vibrating surface are

$$v_r(r,0,t) = 0, \tag{4a}$$

$$v_r(r,h,t) = 0, \tag{4b}$$

$$v_z(r,0,t) = 0, \tag{4c}$$

$$v_z(r,h,t) = dh/dt. \tag{4d}$$

Assuming the inertia force is independent of the film thickness, the inertia term in Eq. (1) can be approximated by the mean volume average across the film thickness

$$\frac{\rho}{h}\int_0^h \left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + v_z\frac{\partial v_r}{\partial z}\right)dz = -\frac{\partial p}{\partial r} + \mu\frac{\partial^2 v_r}{\partial z^2}. \tag{5}$$

From Eqs. (2), (3), (4a)–(4d), and (5) we obtain

$$v_r = \frac{3r}{h^3}(z^2 - hz)\frac{dh}{dt}, \tag{6}$$

$$\frac{\partial p}{\partial r} = \left(\frac{6\mu r}{h^3} \times \frac{dh}{dt}\right) - \frac{9}{10}\frac{r\rho}{h^2}\left(\frac{dh}{dt}\right)^2 + \frac{r\rho}{2h}\frac{d^2h}{dt^2}. \tag{7}$$

If the fluid inertia is taken into account, the boundary conditions for a positively squeeze situation and a negatively squeeze situation are different

$$p(r_0,t) = p_a$$

$$\text{for positively squeeze motion } (dh/dt < 0), \tag{8a}$$

$$p(r_0,t) = p_a - \Delta p$$

$$\text{for negatively squeeze motion } (dh/dt > 0). \tag{8b}$$

A pressure drop Δp occurs when the fluid is sucked into the gap. In other words, the pressure immediately outside of the film edge becomes lower than the ambient pressure P_a by Δp because the in-flow speed at the film edge is the highest near the edge but gradually diminishes away from the edge into the ambient environment. But during the compression cycle, the out-flow air disperses into the open space, so that the boundary condition during compression is the same as the ambient pressure P_a. This boundary condition had been adopted in earlier publications in Refs. 8–10, except their Δp was different from ours.

In order to derive the pressure drop Δp, let us consider the conservation of mechanical energy in fluid,

$$\int_s \left(\frac{\rho}{2}\vec{v}^2 + p\right)dQ = 0, \tag{9}$$

where s is the cross section area of the control volume of the air flow, $\vec{v}$ is the velocity, and Q is the flux. Looking at the r-z cross section of the air film, Eq. (9) may be simplified to

$$\int_0^h v_r\left(\frac{\rho v_r^2}{2} + p\right)dz = \int_0^{h_1} v_1\left(\frac{\rho v_1^2}{2} + p_a\right)dz, \tag{10}$$

$$\Delta p = p_a - p = \frac{\rho}{2Q}\left(\int_0^h v_r^3 dz - \int_0^{h_1} v_1^3 dz\right), \tag{11}$$

$$Q = \int_0^h v_r dz = \int_0^{h_1} v_1 dz. \tag{12}$$

Here h is the height of the flow sheet at the film boundary, h_1 is an effective cross section height of the same flow stream that will is widened outside of the film region, v_1 is the inlet flow velocity with the same velocity profile as v_r and can be derived from Eq. (12). For the same vibration amplitude, smaller h_0 will produce faster flow speed near the exit and there will be more air exchange with outside in terms of relatively air volume, which means that the flow stream will widen more outside of the film, so that h_1 should be inversely proportional to h_0. If h_0 is given, the air speed near the film edge will increase with the vibration amplitude and the radius of the vibrator so that we may use the following expression for the effective h_1:

$$h_1 = h + \alpha\Delta h, \tag{13}$$

where the aspect ratio $\alpha = r_0/h_0$, $\Delta h = \max(h - h_0)$ represents the vibration amplitude at the resonance frequency f.

Solving for the pressure drop Eqs. (11)–(13) and with the consideration of the velocity profile in Eq. (6) we derived the following closed form formula:

$$\Delta p = \frac{27\rho r_0^2}{140 h_0^2}\left[1 - \frac{1}{(1 + \alpha\Delta h/h_0)^2}\right]\left(\frac{dh}{dt}\right)^2. \tag{14}$$

Here, Δp is a function involving essential characteristics of the squeeze film, including film aspect ratio α, the vibration amplitude of the radiator and the squeeze movement velocity dh/dt. It is also easy to see that the coefficient in front of the big brackets in Eq. (14) is proportional to the squeeze number $\sigma = 12\omega\mu r_0^2/p_a h_0^2$ for any given operating frequency.

Integrating the convective inertia Eq. (7) with respect to r and t use the pressure boundary conditions Eqs. (8) and (14), the time-averaged pressure distribution along the radius of the vibrating surface can be obtained. Then, the levitation force can be integrated by time-averaging the pressure distribution on the radiation surface

$$F = \int_0^{r_0} 2\pi r\left[\frac{1}{T}\int_0^T p(r,t)dt\right]dr. \tag{15}$$

Figure 1 shows the levitation force normalized by F_0 calculated using three different models, where F_0 is the levitation force without edge effect ($\Delta p = 0$). In our model, Δp depends on the squeeze number through the aspect ratio. As the squeeze number becomes larger, the edge effect becomes larger and our model approaches the empirical model of Refs. 9 and 10. When the squeeze number becomes smaller,

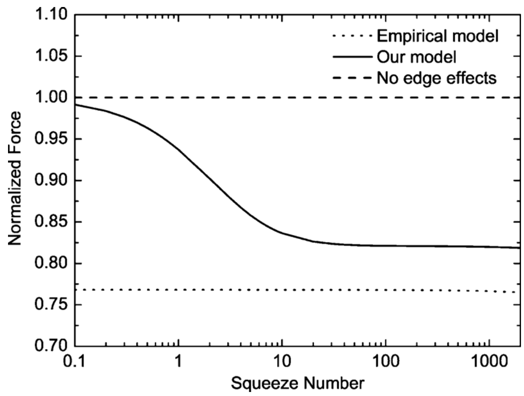

FIG. 1. Comparison of levitation forces of our model with the empirical model and inertia model without edge effect for different squeeze numbers. All values are normalized to the force value without edge effect ($\Delta p=0$).

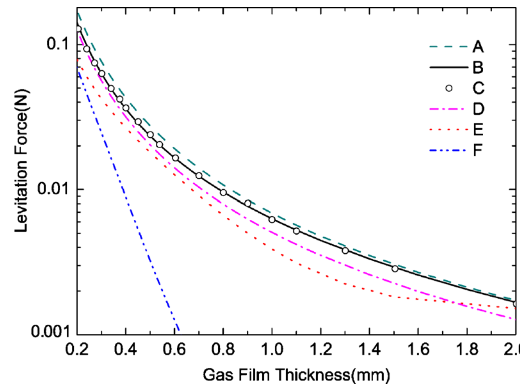

FIG. 2. (Color online) Levitation force predicted by different models and experimentally measured values. A–inertia model with no edge effect; B–our model with gas inertia and aspect ratio dependent edge effect; C–experimental values; D–empirical model (see Refs. 8–10); E–flexural boundary solution (see Ref. 6); F–pistonlike model (see Refs. 3–5).

the edge effect becomes smaller and our model approaches the inertia model without pressure drop. This captured the basic physics because the radial velocity will be close to zero for very small squeeze number (very large h_0) so that the pressure at the film edge will be the ambient pressure ($\Delta p=0$).

For the experimental verification, we built a levitation system in a clean room with a constant temperature environment of 20 °C. The disk vibrator has a radius of 75 mm and works at its second resonant frequency of 19.5 kHz with a power of 80 W. During the force measurement, the average gas film thickness can be changed from 0.2 to 2 mm by a positioning stage moving along the z-direction. The levitation force for different gap size h_0 was measure by a load cell attached to the reflector with an accuracy of 1.5×10^{-3} N. The surface vibration profile of the vibrator was measured by a Polytec scanning vibrometer (PSV-300F-B), which was used as the film boundary shape in the numerical calculations.

Figure 2 shows the comparison between experimental data and calculation results using different theoretical models. In order to provide a global measure, we calculated the relative standard deviation of different theoretical model results from the experimental measured data. As shown in the figure, the piston model (F) is the simplest but has the worst agreement with experimental results (C). The relative standard deviation is as high as 85%. The larger error for smaller squeeze number also comes from the analytical perturbation treatment. The flexural boundary model (E), which solves the problem numerically with realistic boundary shape, shows the correct variation tendency but the relative standard deviation is still too large (~32%). The inertia model without edge effect (A) shows much better agreement but the predicted values are larger than the experimental values with the relative standard deviation of 16.36%. The empirical model of Turns and Kuroda (D), which included inertia and edge effect, made over correction for thicker film situation. Although the empirical model works well for small film thickness (large squeeze number), the theoretical results gradually deviate from experimental values as the film thickness increases with the global relative standard deviation of 15.96%. Our model (B), on the other hand, takes into account the flexural vibrating surface, the gas inertia and a film aspect ratio dependent edge effect, accurately predicted the levitation force for all measured film thickness ranges. The global relative standard deviation is only 2.6%. More importantly, there are no empirical fitting parameters in our model and the model works for a wide range of squeeze numbers. The accuracy of our theoretical model can be clearly seen in Fig. 2. Therefore, our model provides a useful tool for the design of acoustic levitation systems.

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