

Nonlinear restoring forces and geometry influence on stability in near-field acoustic levitation

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Stability is a key factor in near-field acoustic levitation (NFAL), which is a popular method for noncontact transportation of surface-sensitive objects. Since the physical principle of NFAL is based on nonlinear vibration and nonuniform pressure distribution of a plate resonator, traditional linearized stability analysis cannot address this problem correctly. We have performed a theoretical analysis on the levitation stability using a nonlinear squeeze film model including inertia effects and entrance pressure drop, and obtained nonlinear effective restoring force and moment. It was found that the nonuniform pressure distribution is mode-dependent, which determines the stability of the levitation system. Based on the theoretical understanding, we have designed a NFAL resonator with tapered cross section, which can provide higher stability for the levitating object than the rectangular cross-section resonator. © 2011 American Institute of Physics.

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I. INTRODUCTION

In microfabrications, the handling of fragile and surface-sensitive components in microsystems and semiconductor components presents special challenges. Although contact transport systems can provide high load capacity, high speed, and precise positioning, it produced several serious problems, including contact wear, dust generation, mechanical noises and hysteresis.^{1,2} Therefore, in many practical situations, noncontact handling is preferred. Near-field acoustic levitation (NFAL) is one of the most suitable methods for noncontact handling. Solid materials, including insulators, conductors, magnetic or nonmagnetic materials, can all be manipulated by NFAL. In NFAL, the object is levitated above the acoustic wave radiation surface at a height much smaller compared to the wavelength. There are many experimental³⁻⁵ and theoretical studies on levitation force in NFAL. For example, Hasegawa *et al.*⁶ calculated the acoustic radiation pressure on a small rigid sphere situated on the axis of a circular piston vibrator, Chu *et al.*⁷ summarized the models and existing problems in acoustic radiation pressure studies, Minikes and Bucher⁸ solved the coupled dynamic problem of a vibrating piezoelectric disk, which generates an air squeeze film. In these theoretical treatments, the radiator was assumed to be a rigid piston with uniform vibration amplitude. However, in reality, because the radiator may operate at different resonance modes, surface vibration amplitudes are generally nonuniform. Therefore, further theoretical investigation is needed to clarify such vibration surface nonuniformity.⁹ It was demonstrated recently that the theoretical predictions can match the measured results better if the flexural mode shape is taken into consideration when solving the squeeze film problem.^{10,11}

There are two important aspects in NFAL. One is the magnitude of the levitation force and the other is the levitation stability. Experimental measurements revealed that the stability of an acoustically levitated object is highly dependent on the vibration amplitude distribution of the radiation surface.¹² Ide *et al.*¹³ demonstrated experimentally a linear bearing with an angular cross section to achieve better stability. Ueha *et al.*¹⁴ used a T-shape plate in levitation experiment, which provided stable noncontact transportation. Matsuo *et al.*¹² and Hu *et al.*¹⁵ treated the effect of the acoustic viscous force on the stability as a linear block-spring system with an equivalent spring constant. However, because of the linear approximation of a block-spring system, this method can only treat small amplitude oscillation but not the true nonuniform pressure distribution and its relationship with levitation stability. Moreover, the inclination and restoring moment were not included. Without quantitative modeling of the pressure distribution, a stability evaluation is restricted only to the acoustic field generated by a flat radiation surface. In practical applications, the shape of the vibrators may change to fit the needs, such as V-shaped¹³ and T-shaped¹⁴ vibrators. Therefore, a quantitative physical method for stability analysis of arbitrary shaped resonators is urgently needed for the optimization of NFAL.

In this work, we use a nonlinear squeeze film model, which includes inertia effects and entrance pressure drop,¹⁶ to calculate the pressure distribution for different operating modes and resonator plates with different cross sections. Based on the nonlinear vibration and nonuniform pressure distribution, we have quantitatively studied the nonlinear levitation system. Stability analysis under a small disturbance was carried out through a time-averaged local potential. Both eccentricity and inclination cases are discussed, and the restoring forces as well as the corresponding moments have been calculated. Our results showed obvious nonlinearity of the equivalent spring coefficient, which could not be

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addressed in previous linear models. The effect of cross section on stability behavior can also be studied using our method. By comparing the restoring force against off-center motion and the restoring moment against inclination, the tapered cross section vibrator showed much higher levitation stability than a rectangular cross section vibrator. This theoretical result explained the former experiments^{12,14,15} that demonstrated a tapered vibrator provided more stable levitation. Our theoretical approach provides a useful tool for the design and optimization of a near-field acoustic levitation system for noncontact transportation and handling of surface-sensitive objects.

II. THEORETICAL MODEL

A. Governing equation

Figure 1 shows the cross section of an axisymmetric half structure of the NFAL system. A vibration plate with a radius r_0 vibrates at a resonance frequency f and a reflector of the same dimension is placed above at a distance h_0 . The thin layer of air between the vibrator and the reflector is conventionally called gas squeeze film, which induces the levitation phenomenon. The squeeze film thickness h_0 is in the order 10^{-4} m, and is generally two orders of magnitude smaller than the radius r_0 . The total air inertia force includes the local inertia force and the convective force. Based on the Stokes microcontinuum theory,¹⁷ considering an axisymmetric radial laminar flow, the momentum equation for Newtonian fluids with inertia terms is given by

$$\rho \left(\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + v_z \frac{\partial v_r}{\partial z} \right) = -\frac{\partial p}{\partial r} + \frac{\partial \tau_{rz}}{\partial z}. \quad (1)$$

Where $\tau_{rz} = \mu \frac{\partial v_r}{\partial z}$ is the shear stress, the viscosity is considered only in the thickness direction,¹⁷ p , ρ and μ are gas pressure, density and viscosity, respectively; v_r is the velocity component along r .

In our case, the gas film thickness is much much smaller than the radius. Therefore, it is reasonable to assume that the inertia force is independent of the film thickness. The inertia term in (1) can be approximated by the mean volume average across the film thickness, the pressure distribution in an axisymmetric radial squeeze film can be expressed as:

$$\frac{\partial p}{\partial r} = \frac{6\mu r}{h^3} \frac{dh}{dt} - \frac{9}{10} \frac{r\rho}{h^2} \left(\frac{dh}{dt} \right)^2 + \frac{r\rho}{2h} \frac{d^2 h}{dt^2}. \quad (2)$$

Equation (2) represents the differential relationship among the pressure, film thickness, radius, and time. With a proper

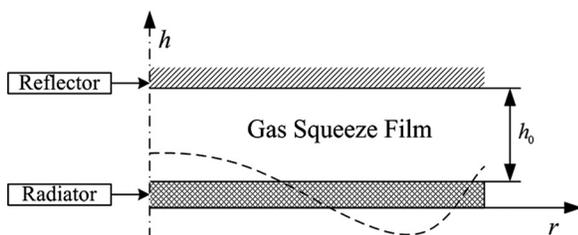


FIG. 1. Schematic plot of near-field acoustic levitation and the coordinate system. Dashed line: illustration of the vibration shape of the vibrator surface.

boundary pressure at the edge of the film, one can get the pressure distribution pattern by solving the differential equation.

When the fluid inertia is taken into consideration, the boundary conditions for positively squeezed and negatively squeezed situations are different:

$$p(r_0, t) = p_a \text{ for positive squeeze motion } (dh/dt < 0) \quad (3a)$$

$$p(r_0, t) = p_a - \Delta p \text{ for negative squeeze motion } (dh/dt > 0). \quad (3b)$$

A pressure drop Δp occurs when the fluid is sucked into the gap between the two disks. In other words, the pressure at the edge becomes lower than the ambient pressure by Δp . The entrance pressure drop based on the energy conservation theory is given by¹⁶

$$\Delta p = \frac{27\rho r_0^2}{140h_0^2} \left(1 - \frac{1}{(1 + \alpha\Delta h/h_0)^2} \right) \left(\frac{dh}{dt} \right)^2. \quad (4)$$

Integrating the convective inertia Eq. (2) with respect to r and t using the pressure boundary conditions in Eqs. (3a) and (3b), the time-averaged pressure distribution along the radius of the vibrating surface can be obtained. Then, the levitation force is obtained by time averaging the pressure distribution on the radiation surface:

$$F = \int_0^{r_0} 2\pi r \left(\frac{1}{T} \int_0^T p(r, t) dt \right) dr. \quad (5)$$

B. Local potential and restoring force

When the center of the levitated object is not concentric with the pressure field, there is a horizontal movement due to the restoring force of NFAL and the inertia of the object. Once the levitated object is slightly eccentric from the vibrator, the acoustic potential is no longer axisymmetric. Therefore, we need to analyze the local potential change caused by the off-center motion of the object. The moving direction of the levitated object can be predicted from the calculated distribution of a dimensionless time-averaged local potential¹⁸

$$\tilde{U} = \frac{1}{3} \tilde{p}^2 - \frac{1}{2} \tilde{v}^2, \quad (6)$$

where $\tilde{p} = \bar{p}/\rho c^2$, c is the speed of sound in air, $\tilde{v} = \bar{v}/c$, and \bar{p} and \bar{v} are functions of position, which denote the root mean-square pressure and velocity, respectively.

The dimensionless nonlinear force acting on the levitated object can then be obtained from the local potential gradient

$$\tilde{F}_{\text{res}} = -\text{grad}(\tilde{U}). \quad (7)$$

The minus sign denotes that when eccentricity occurs, the levitated object will move into the region with lower potential.

C. Restoring moment

Another type of disturbance is the inclination of the levitated object. Under a small inclination, the film thickness may be asymmetric, which gives rise to asymmetric pressure distribution. As a result, Eq. (2) can be rewritten as

$$\frac{\partial p(r, \theta)}{\partial r} = \frac{6\mu r}{h^3(r, \theta)} \frac{dh(r, \theta)}{dt} - \frac{9}{10} \frac{r\rho}{h^2(r, \theta)} \left(\frac{dh(r, \theta)}{dt} \right)^2 + \frac{r\rho}{2h(r, \theta)} \frac{d^2h(r, \theta)}{dt^2} \quad (8)$$

where r and θ are polar coordinates.

Considering a simple inclination without eccentricity, the center of the levitated object and the radiator are homocentric, so that there is no horizontal force acting on the object. When there is an inclination around the axis at $\theta = \pi/2$, the restoring moment may be calculated as

$$M = \int_0^{2\pi} \int_0^R -p(r, \theta) r^2 \cos \theta dr d\theta. \quad (9)$$

III. AMPLITUDE DISTRIBUTION PATTERN AND LEVITATION FORCE

In NFAL, the vibrator can operate at different resonance modes, which leads to various displacement patterns. Most existing theoretical models ignored the nonuniformity of the kinetic boundary condition by treating the surface as a rigid plane. We found that the nonuniform surface vibration profile has significant influence on levitation characteristics in the gas film and, hence, cannot be ignored. In fact, we may design the vibrator to produce a desired nonuniform vibration amplitude distribution for better stability. Ueha *et al.*¹⁴ had shown experimentally that a tapered plate could produce more stable levitation.

In this work, we have studied vibrators with rectangular and tapered cross sections. The aluminum vibrator with a rectangular cross section is a round disk with a thickness of 14 mm and a radius of 75 mm. The layout of the vibrator with a tapered cross section is shown in Fig. 2. The vibration displacements in the first three modes simulated by the ANSYS finite element analysis package are shown in Fig. 3. In each mode, we design the two vibrators to generate the same levitation forces by adjusting the displacement amplitude, so that the following stability comparison can be done under the same load capacity. The calculation parameters are summarized in Table I together with the total levitation force for each mode. The radius of each nodal circle in the tapered plate is larger than that of the rectangular plate. Because the displacement amplitudes are kept in the same order of mag-

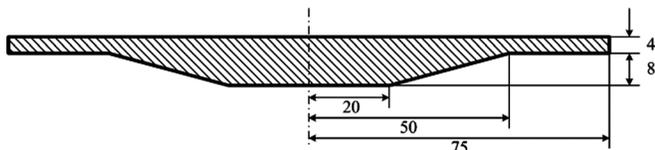


FIG. 2. Layout of the tapered vibrator (unit = mm).

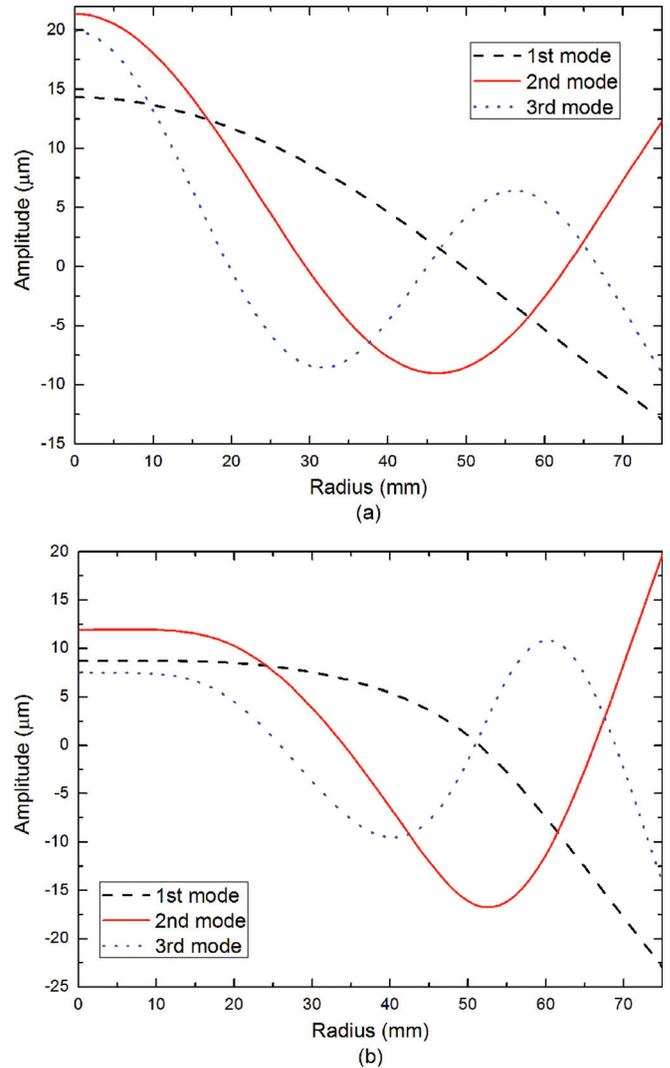


FIG. 3. (Color online) Displacement as a function of the radius for each mode. (a) rectangular cross-section plate resonator; (b) tapered cross-section plate resonator.

nitude for all modes, the 1st mode will generate less time-averaged pressure so that it is not efficient for practical applications, while the 3rd mode needs much large power to excite. Therefore, the 2nd mode is used as a compromise, which can produce considerable levitation force with relatively high electromechanical energy conversion efficiency.

IV. STABILITY ANALYSIS

The stability of levitation is an important requirement in practical applications. There are two kinds of disturbances: (1) the object is off center with respect to the circular symmetric acoustic field; (2) the object is inclined from the horizontal plane. We will analyze each case separately based on Eqs. (7) and (9).

A. Local potential and restoring force against eccentric motion

As the levitated object becomes eccentric with respect to the vibrator, the pressure distribution and the time-averaged potential will be changed. Figure 4 shows the contours

TABLE I. Detailed parameters of plates with tapered and rectangular cross sections.

Cross section	Frequency (Hz)	Levitation distance (μm)	Radius of 1 st nodal circle (mm)	Radius of 2 nd nodal circle (mm)	Radius of 3 rd nodal circle (mm)	Levitation force (N)
Tapered	3274	300	51.48	—	—	0.05
	11713	300	34.09	66.22	—	0.5
	25141	300	25.88	50.90	68.93	1
Rectangular	5261	300	49.65	—	—	0.05
	19522	300	29.50	62.80	—	0.5
	38075	300	19.69	44.96	66.84	1

of the dimensionless time-averaged local potential caused by eccentricity. Both rectangular and tapered cross section vibrators are excited in the 2nd mode. The analyzed eccentric motion of the levitation plate is to the right. The inner level curves represent higher potential; the difference between adjacent equipotential lines in each figure is the same but their values differ from figure to figure because there are 10 lines in each figure but the maxima are different. For both cases, the time-averaged potential distribution on the right half is higher than that in the left half, producing a total restoring force for the eccentric motion. In Fig. 4, the plots on the left are for rectangular cross sections while those on the right are for tapered cross section resonators. The potential generated by the same eccentric displacement is always higher in the tapered case. As shown in Fig. 5, the centripetal restoring force increases nonlinearly with the off-center distance. At the same eccentricity d , the tapered cross

section vibrator generates a larger restoring force than the rectangular cross section vibrator, which indicates that the tapered vibrator provides more stable NFAL levitation.

B. Restoring moment by inclination

Another disturbance is the inclination from the horizontal plane. This corresponds to a small rotation around an axis along the diameter of the vibrator. For a small inclination, we can calculate the pressure distribution and restoring moment from Eq. (9). The restoring moment comes from the asymmetric pressure distribution due to the asymmetric air thickness. Figure 6 shows the calculated restoring moment as a function of the inclined angle. Because the levitating object should not touch the surface of the vibrator and the ratio of thickness over radius is very small, we only calculated a very small tilting angle. Even with such a small angle of inclination, one could see clearly the restoring moment toward the horizontal position. One can also see that the restoring moment increases very quickly as the inclination angle increases. In addition, the tapered vibrator offers a larger restoring moment to push the object back to the horizontal equilibrium position.

V. SUMMARY AND CONCLUSIONS

Near-field acoustic levitation is one of the most popular methods for noncontact manipulation of surface-sensitive objects. By using nonuniform vibration displacement distribution as a kinematic boundary constraint, we have developed a

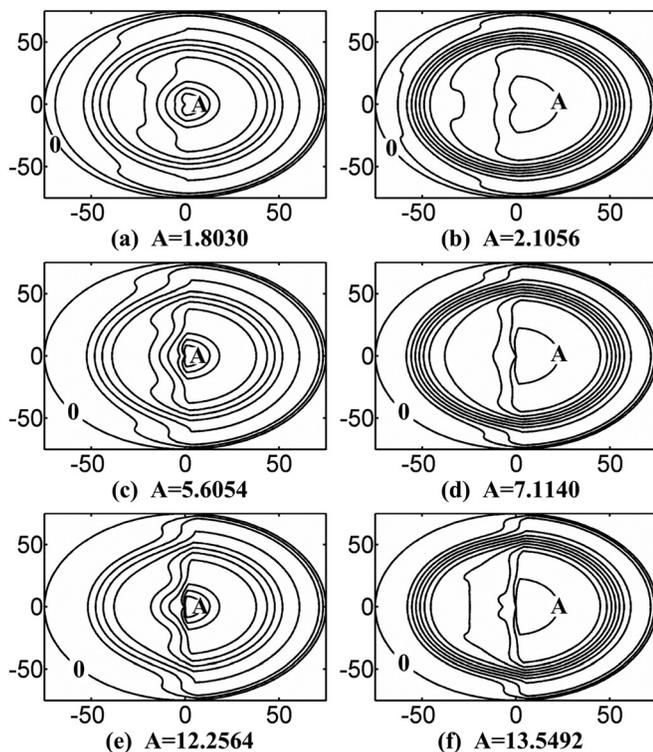


FIG. 4. Contours of the time averaged dimensionless potential generated by rectangular and tapered cross section vibrators: (a) Rectangular, $d = 1$ mm; (b) Tapered, $d = 1$ mm; (c) Rectangular, $d = 7$ mm; (d) Tapered, $d = 7$ mm; (e) Rectangular, $d = 13$ mm; (f) Tapered, $d = 13$ mm. The outermost line is set as potential 0, while A is the highest potential value of the innermost contour.

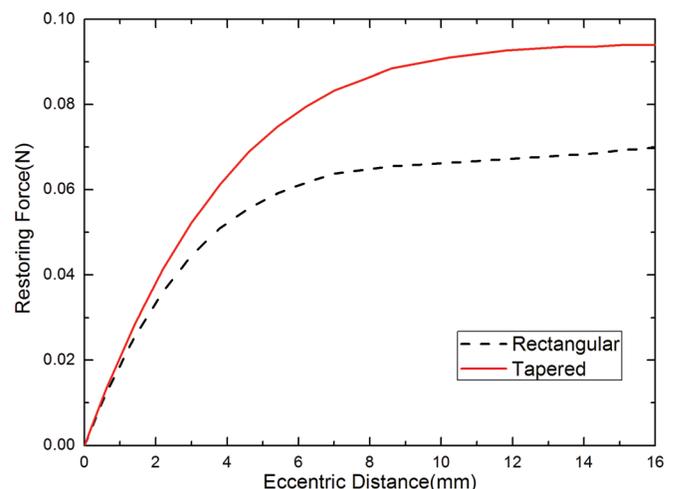


FIG. 5. (Color online) Restoring force vs eccentric distance.

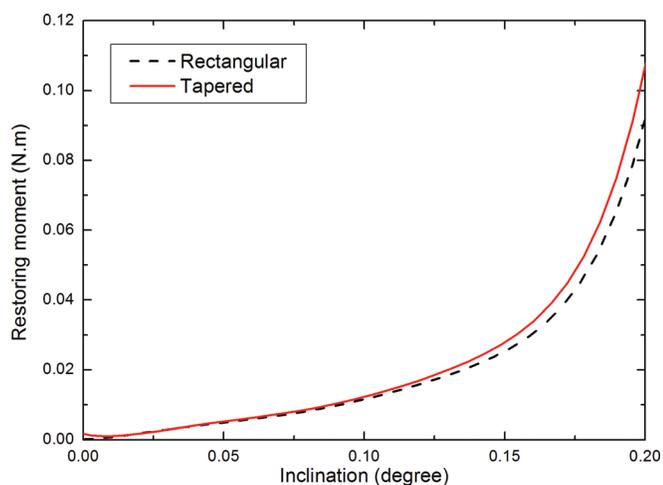


FIG. 6. (Color online) Restoring moment vs inclination angle.

theoretical method to calculate nonuniform pressure distribution in a thin air film and the total levitation force produced. Our model was derived based on microcontinuum theory by including gas inertia terms and an entrance pressure drop.¹⁶ Based on the nonlinear pressure distribution, a quantitative method for the stability evaluation has been proposed in this work to describe the nature of stability behaviors during levitation, and the model is valid for any arbitrary nonuniform acoustic field.

As we have proven in the finite element analysis, the surface displacement distributions generated by different modes of the same plate are quite different. We also found that the cross section of the vibrating plate has significant influence on the surface displacement distribution. Vibrators with rectangular cross sections and tapered cross sections have been investigated in our levitation stability study and we showed that the vibrator with a tapered cross section can generate a larger restoring force against eccentric motion or inclination in comparison with a vibrator having a rectangular cross section. In other words, a tapered vibrator design can improve the stability of an acoustically levitated object, which explains the experimental results of Refs. 14 and 15. The nonlinear nature resulting from the eccentric motion and inclination is obvious in Figs. 5 and 6, hence, one could not

use an equivalent spring constant to describe the restoring forces and the restoring moments. Compared with former studies using a linear block-spring system approximation, our method captures the nonlinear features of the restoring forces. In principle, our theoretical procedure can be used in resonator design optimization for specified levitation characteristics in the NFAL.

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- ¹G. Reinhart and J. Hoepfner, *CIRP Ann.* **49**, 5 (2000).
- ²D. Koyama, T. Ide, J. R. Friend, K. Nakamura, and S. Ueha, *IEEE Trans. Ultrason. Ferroelectr. Freq. Control* **54**, 597 (2007).
- ³Sadayuki Ueha, *Proceedings of IEEE Ultrasonics Symposium*, **1**, 513 (2001).
- ⁴Y. Hashimoto, Y. Koike and S. Ueha, *J. Acoust. Soc. Jpn. E* **16** (3), 189 (1995).
- ⁵T. Ide, J. R. Friend, K. Nakamura, and S. Ueha, *Proceedings of the World Congress on Ultrasonics Institut, Biomedical des Cordeliers Universite Pierre et Marie Curie, Paris, France, 7-10 September 2003*, p. 1577.
- ⁶T. Hasegawa, T. Kido, S. Takeda, N. Inoue, and K. Matsuzama, *J. Acoust. Soc. Am.* **88**, 1578 (1990).
- ⁷B. Chu, R. E. Apfel, *J. Acoust. Soc. Am.* **72**, 1673 (1982).
- ⁸A. Minikes and I. Bucher, *J. Sound Vib.* **263**, 241 (2003).
- ⁹H. Nomura and T. Kamakura, *J. Acoust. Soc. Am.* **111**, 1578 (2002).
- ¹⁰P. Liu, J. Li, H. Ding, and W. Cao, *IEEE Trans. Ultrason. Ferroelectr. Freq. Control* **56**, 2679 (2009).
- ¹¹J. Li, P. Liu, paper presented at the 5th International Workshop on Piezoelectric Materials and Applications in Actuators, 6–10 October 2008, Pennsylvania State University, College Park, PA, USA.
- ¹²E. Matsuo, Y. Koike, K. Nakamura, S. Ueha, and Y. Hashimoto, *Ultrasonics* **38**, 60 (2000).
- ¹³T. Ide, J. Friend, K. Nakamura, and S. Ueha, *Sensors and Actuators A: Physical* **135** (2), 740 (2007).
- ¹⁴S. Ueha, Y. Hashimoto and Y. Koike, *Ultrasonics* **38**, 26 (2000).
- ¹⁵J. Hu, K. Nakamura, and S. Ueha, *IEEE Trans. Ultr. Ferr. Freq. Contr.*, **50** (2), 117 (2003).
- ¹⁶J. Li, W. Cao, P. Liu, and H. Ding, *Appl. Phys. Lett.* **96**, 243507 (2010).
- ¹⁷V. K. Stokes, *Phys. Fluids* **9**, 1709 (1966).
- ¹⁸M. Barmatz and P. Collas, *J. Acoust. Soc. Am.* **77** (3), 928 (1985).