

## Stress and electric displacement distribution near Griffith's type III crack tips in piezoceramics

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The inhomogeneous distributions of internal shear stress and electrical field induced by external shear stress or applied electric field around a Griffith's type III crack tip in ferroelectric ceramics have been analyzed. For a linear system, the stress and the electric displacement intensity factors,  $K_{III}$  and  $K_{IV}$ , respectively, can be expressed in simple analytic forms which account for both electric and mechanical contributions.

The performance of piezoceramics is altered by the presence of cracks, cuts, narrow cavities and similar flaws which may propagate under certain conditions causing eventual destruction of a body as a whole. Today, the problem of mechanical reliability of ferroelectric ceramics becomes increasingly important as the materials are used in more and more sophisticated areas. An updated review about the fracture problem in ferroelectric ceramics was given by Freiman [1]. More recent studies [1-3] show that the mechanical behavior of poled PZT ceramics is greatly affected by the external force induced inhomogeneous distribution of internal stress, and the type of cracks. There exist both electric field and mechanical stress concentration near crack tips, which induces crack propagation and incompatible elastic deformation in ferroelectric ceramics. In usual non-ferroelectric brittle ceramics, the stress intensity factor  $K_J$  ( $J=I, II, III$ ) is related to the stress  $\sigma$  by [4-6]

$$K_J = \sigma Y \sqrt{a}, \quad (1)$$

where  $a$  is the crack length,  $Y$  is the shape factor of a specimen. For ferroelectric ceramics the expression of eq. (1) needs to be modified due to the piezoelectric effect. Parton et al. [7], have given a general description of the electroelastic plane problem for a piezoelectric medium containing a rectilinear crack, and analyzed in detail the Griffith's type I crack. In this Letter, we discuss the influence of Griffith's type

III crack on electric and mechanical properties in piezoceramics. Relatively simple expressions of the stress and electrical displacement intensity factors  $K_{III}$  and  $K_{IV}$  have been derived, which can be used to evaluate the mechanical behavior of piezoceramics.

The results derived here may be very helpful in some specific applications, such as transducers with thickness shear vibration mode, mismatch and incompatible deformation between ferroelectric ceramics and substrates under external shearing force in multilayer devices, composite devices and electronic packages.

If a ceramic sample is poled along the  $x_3$  axis, its mechanical, dielectric and piezoelectric properties are described by five elastic moduli, two dielectric and three piezoelectric coefficients. In Voigt notation, the constitutive equations may be written as [7,8]

$$\sigma_{11} = c_{11}^E s_{11} + c_{12}^E s_{22} + c_{13}^E s_{33} - e_{31} E_3, \quad (2a)$$

$$\sigma_{22} = c_{12}^E s_{11} + c_{22}^E s_{22} + c_{23}^E s_{33} - e_{31} E_3, \quad (2b)$$

$$\sigma_{33} = c_{13}^E (s_{11} + s_{22}) + c_{33}^E s_{33} - e_{33} E_3, \quad (2c)$$

$$\sigma_{23} = 2c_{44}^E s_{23} - e_{15} E_2, \quad (2d)$$

$$\sigma_{13} = 2c_{44}^E s_{13} - e_{15} E_1, \quad (2e)$$

$$\sigma_{12} = (c_{11}^E - c_{12}^E) s_{12}, \quad (2f)$$

$$D_1 = \epsilon_{31} E_1 + 2e_{15} s_{13}, \quad (2g)$$

$$D_2 = \epsilon_{11} E_2 + 2e_{35} s_{23}, \quad (2h)$$

$$D_3 = \epsilon_{33} E_3 + e_{31} (s_{11} + s_{22}) + e_{33} s_{33}, \quad (2i)$$

where  $s_{ij} = \frac{1}{2}(\partial u_j / \partial x_i + \partial u_i / \partial x_j)$  is the elastic strain component;  $u_j$  ( $j = 1, 2, 3$ ) is the elastic displacement field,  $E_j = -\partial \phi / \partial x_j$  is the electric field strength (the depolarization field is not included), and  $\phi$  is the electric potential,  $e_{31}$ ,  $e_{33}$  and  $e_{15}$  are the piezoelectric coefficients,  $c_{11}^E$ ,  $c_{12}^E$ ,  $c_{13}^E$ ,  $c_{33}^E$  and  $c_{44}^E$  are the elastic moduli at constant electric field,  $\epsilon_{11}$  and  $\epsilon_{33}$  are the dielectric permittivities at constant strain.

For a system under longitudinal shear stress (see fig. 1), the so-called antiplane problem, we have the following conditions [5,6]:

$$u_1 = u_2 = 0, \quad (3a)$$

$$u_3 = u_3(x_1, x_2), \quad (3b)$$

$$\phi = \phi(x_1, x_2). \quad (3c)$$

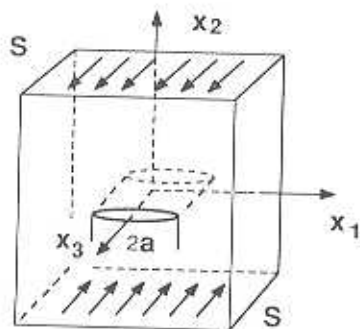


Fig. 1. The Griffith's type III crack in a piezoelectric ceramic.  $x_3$  is the poling direction. Shear stress is applied at  $x_2 = \pm L/2$  surface in  $x_3$  axis direction. The electric field is applied along the direction of the  $x_3$  axis.

Eqs. (3a)–(3c) imply that we have a two-dimensional problem and

$$s_{11} = s_{22} = s_{33} = s_{12} = 0, \quad E_3 = 0. \quad (4)$$

Substituting eqs. (4) into eqs. (2) gives

$$\sigma_{23} = 2c_{44}^E s_{23} - e_{15} E_2, \quad (5a)$$

$$\sigma_{13} = 2c_{44}^E s_{13} - e_{15} E_1, \quad (5b)$$

$$D_1 = \epsilon_{11} E_1 + 2e_{15} s_{13}, \quad (5c)$$

$$D_2 = \epsilon_{11} E_2 + 2e_{15} s_{23}. \quad (5d)$$

Considering a system under both mechanical stress and electric field, the Euler and Maxwell equations have the following forms [5–7]:

$$\partial \sigma_{32} / \partial x_2 + \partial \sigma_{31} / \partial x_1 = 0, \quad (6a)$$

$$\partial D_1 / \partial x_1 + \partial D_2 / \partial x_2 = 0. \quad (6b)$$

Substituting eqs. (5) into eqs. (6) gives

$$c_{44} \nabla^2 u_3 + e_{15} \nabla^2 \phi = 0, \quad (7a)$$

$$e_{15} \nabla^2 u_3 - \epsilon_{11} \nabla^2 \phi = 0. \quad (7b)$$

Generally speaking, the determinant of eqs. (7a) and (7b) is non-zero, i.e.

$$\Delta = \begin{vmatrix} c_{44} & e_{15} \\ e_{15} & -\epsilon_{11} \end{vmatrix} \neq 0.$$

One can easily verify this from the data in table 1 which lists the parameters for the most widely used piezoceramics PZT 65/35, and PZT-4. Therefore, from eqs. (7) one has

$$\begin{pmatrix} \nabla^2 u_3 \\ \nabla^2 \phi \end{pmatrix} \equiv \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \quad (8)$$

We choose the  $x_2 = 0$  to be the reference plane for electric potential [ $\phi(x_2 = 0) = 0$ ]. Therefore, the two-

Table 1

The elastic, dielectric and piezoelectric coefficients for PZT 65/35 from ref. [10] and PZT-4 from ref. [11]

Piezoelectric ceramic PZT 65/35		Piezoelectric ceramic PZT-4	
$c_{11}^E = 1.594 \times 10^{11} \text{ N/m}^2$	$e_{31} = -6.127 \text{ C/m}^2$	$c_{11}^E = 13.9 \times 10^{10} \text{ N/m}^2$	$e_{31} = -5.2 \text{ C/m}^2$
$c_{12}^E = 7.385 \times 10^{10} \text{ N/m}^2$	$e_{33} = 10.71 \text{ C/m}^2$	$c_{12}^E = 6.78 \times 10^{10} \text{ N/m}^2$	$e_{33} = 15.1 \text{ C/m}^2$
$c_{13}^E = 1.261 \times 10^{11} \text{ N/m}^2$	$e_{15} = 8.387 \text{ C/m}^2$	$c_{13}^E = 17.43 \times 10^{10} \text{ N/m}^2$	$e_{15} = 12.7 \text{ C/m}^2$
$c_{44}^E = 3.89 \times 10^{10} \text{ N/m}^2$	$\epsilon_{11} = 5.66 \times 10^{-9} \text{ F/m}$	$c_{33}^E = 11.5 \times 10^{10} \text{ N/m}^2$	$\epsilon_{11} = 6.45 \times 10^{-9} \text{ F/m}$
$c_{66}^E = 4.276 \times 10^{10} \text{ N/m}^2$	$\epsilon_{33} = 2.243 \times 10^{-9} \text{ F/m}$	$c_{44}^E = 2.56 \times 10^{10} \text{ N/m}^2$	$\epsilon_{33} = 5.62 \times 10^{-9} \text{ F/m}$
		$c_{66}^E = 3.06 \times 10^{10} \text{ N/m}^2$	

dimensional problem (on  $x_3=0$  plane) contains one mirror symmetry line ( $x_2$  axis) and the inversion symmetry, thus we only need to study this problem in the first quadrant of the  $x_3=0$  plane ( $x_1 > 0$ ;  $x_2 > 0$ ), as illustrated in fig. 2. On the  $x_1$  axis we have two boundary conditions for  $x_1 > a$ :

$$u_3 = 0, \quad \text{when } |x_1| > a; \quad (9a)$$

$$\varphi = 0, \quad \text{when } |x_1| > a. \quad (9b)$$

In addition, the contour of the crack is free of mechanical load and the crack may be considered as a vacuum or air-filled cavity. Since the value  $\epsilon_0/\epsilon_1$  is very small ( $\epsilon_0$  is vacuum permittivity;  $\epsilon_1$  is ferroelectric ceramics permittivity), the following boundary conditions also hold on the  $x_1$  axis [7,8]:

$$\sigma_{32} = 0, \quad \text{when } |x_1| \leq a; \quad (10a)$$

$$D_2 = 0, \quad \text{when } |x_1| \leq a. \quad (10b)$$

The strain and the applied electric field strength at  $x_2=L/2$  are  $S$  and  $E_0$ , respectively. Hence, the boundary conditions at  $x_2=L/2$  are

$$-\partial\varphi/\partial x_2|_{x_2=L/2} = E_0, \quad (11a)$$

$$\partial u_3/\partial x_2|_{x_2=L/2} = 2S. \quad (11b)$$

As shown in fig. 1,  $L$  is the sample dimension in  $x_2$  direction and  $2a$  is the width of the cut in  $x_1$  direction with  $L \gg 2a$ .

The Laplace equation (8) can be solved by using Fourier transformation technique [6]. The solutions for the displacement  $u_3$  and the electric potential  $\varphi$ , which satisfy the boundary conditions of eqs. (10) with arbitrary larger  $L$ , are

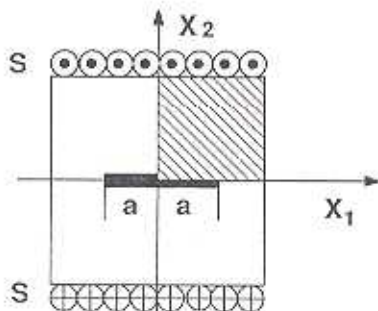


Fig. 2. The  $x_3=0$  plane which is a cross section of fig. 1. The  $x_2$  axis is a mirror symmetry line.

$$u_3(x_1, x_2) = 2S \left[ x_2 + \frac{2}{\pi} \int_0^{\infty} A(\xi) e^{-\xi x_2} \cos(\xi x_1) d\xi \right], \quad (12a)$$

$$\varphi(x_1, x_2) = -E_0 \left[ x_2 + \frac{2}{\pi} \int_0^{\infty} B(\xi) e^{-\xi x_2} \cos(\xi x_1) d\xi \right]. \quad (12b)$$

From eqs. (12) and (10), the unknown functions  $A(\xi)$  and  $B(\xi)$  can be determined by the Daul integral equations:

$$\frac{2}{\pi} \int_0^{\infty} \xi [A(\xi), B(\xi)] \cos(\xi x_1) d\xi = 1, \quad 0 \leq x_1 \leq a; \quad (13a)$$

$$\int_0^{\infty} [A(\xi), B(\xi)] \cos(\xi x_1) d\xi = 0, \quad x_1 > a. \quad (13b)$$

The solutions of eqs. (13a) and (13b) are

$$A(\xi) = B(\xi) = (\pi a/2) \xi^{-1} J_1(a\xi), \quad (14)$$

where  $J_1(a\xi)$  is the first-order Bessel function.

Finally, in order to evaluate the stress and the electrical displacement intensity factors,  $K_{III}$  and  $K_{IV}$ , we derive the distributions of stress and electric field in the vicinity of a crack tip along the  $x_1$  axis,

$$\sigma_{32}(x_1, 0) = 0, \quad |x_1| \leq a; \\ = \frac{d_1 |x_1|}{\sqrt{x_1^2 - a^2}}, \quad |x_1| > a, \quad (15a)$$

$$D_2(x_1, 0) = 0, \quad |x_1| \leq a; \\ = \frac{d_2 |x_1|}{\sqrt{x_1^2 - a^2}}, \quad |x_1| > a. \quad (15b)$$

Here,

$$d_1 = 2c_{44}^E S - e_{15} E_0, \quad (16a)$$

$$d_2 = \epsilon_{11} E_0 + 2e_{15} S; \quad (16b)$$

$d_1$  and  $d_2$  are the total stress and electric displacement at  $x_2=L/2$ , respectively. Obviously, from eqs. (15) both stress and electric displacement fields diverge at the crack tip and decrease toward asymptotic values  $d_i$  for  $|x_1| \gg a$ . According to the defi-

nitions of the stress and the electric displacement intensity factors [6,7], we have

$$K_{III} = \lim_{x \rightarrow a} \sqrt{2\pi(x_1 - a)} \sigma_{23}(x_1, 0) = \sqrt{\pi a} d_1, \quad (17a)$$

$$K_{IV} = \lim_{x \rightarrow a} \sqrt{2\pi(x_1 - a)} D_2(x_1, 0) = \sqrt{\pi a} d_2. \quad (17b)$$

Eqs. (17a) and (17b) are analogous to the fracture conditions known for anisotropic materials [5,12], however, here the coefficients  $d_1$  and  $d_2$  contain both mechanical and electric contributions, reflecting the characteristic of piezoelectric materials. It is interesting to note that these two contributions can either be additive or cancel each other in one of the two intensity factors, depending on the relative direction between the applied electric field and the external mechanical load. In other words, the applied electric field (mechanical stress) can either weaken or enhance the stress (electric displacement) concentration in a piezoelectric material. The overall strength of a material is characterized by the critical values of the two intensity factors,  $K_{III}^c$  and  $K_{IV}^c$ . However, the additive nature of the two contributions in at least one of the intensity factors does not imply that a piezoelectric material is weaker than a non-piezoelec-

tric material, which also depends on the magnitude of  $K_{III}^c$  and  $K_{IV}^c$ .

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