

Electromechanical coupling coefficient of an ultrasonic array element

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One of the most important parameters for characterizing piezoelectric materials is the so-called electromechanical coupling coefficient, k , which describes the electromechanical coupling strength. Although this parameter should be an intrinsic material parameter, it appears to depend on the aspect ratio of the resonator. There are three different values defined for three extreme geometries, k_{33} , k'_{33} , and k_t , and they differ by more than 50%. Unfortunately, these three values cannot describe resonators of general geometries and also create conceptual confusion. Here, we provide a unified formula that will accurately describe the coupling coefficient of rectangular slender bar transducer array element with any aspect ratio. © 2006 American Institute of Physics.

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I. INTRODUCTION

In linear array or phased array transducers for ultrasonic image, each vibrating element is in the form of a rectangular slender bar. The dimension along the poling direction is determined by the operating frequency, while the dimension along the array and perpendicular to the poling direction should be narrow for better directivity of the produced ultrasonic beam. The other dimension perpendicular to the above two directions is usually large in order to increase the total element radiating area for meeting the power output requirement. In order to design transducers with broader bandwidth for better resolution, it is important for each of the element to have high electromechanical coupling coefficient.¹

There are two factors contributing to the effective electromechanical coupling coefficient of a rectangular slender bar resonator. One is the coupling coefficient of the piezoelectric material, which is intrinsic. The other is the aspect ratio of the resonator, which is extrinsic, depending on the design. It is well known that the maximum electromechanical coupling coefficient for the poling direction vibration is k_{33} , which is for the case of a thin and long rod poled along the long dimension, while the smallest is k_t , which is for a thin plate poled along the surface normal. In the case of Pb(Zr,Ti)O₃ (PZT) ceramics, k_{33} is as high as 70% while k_t is only about 48%. Because the rectangular slender bar element must keep one of the lateral dimensions large, the maximum coupling coefficient is k'_{33} (Note: k'_{33} is called k_{33}^w in the IEEE standard for piezoelectricity²), which is smaller than k_{33} . In the case of PZT ceramics, k'_{33} is about 65%.

There have been numerous methods developed to maximize the electromechanical coupling coefficient of resonators. The 1-3 and 2-2 type piezoelectric-epoxy composites are successful examples.³⁻⁶ Instead of using a thin plate

mode that would have a low coupling coefficient k_t , the 1-3 composite plate uses many thin rods embedded in a polymer resin so that the plate vibration is changed into many thin rod vibrations. The effective electromechanical coupling coefficient is therefore being changed from k_t into an effective value very close to k_{33} . In order to increase the effective electromechanical coupling of a rectangular slender bar array element, the element is usually subdiced to make 2-2 type composite, which can convert k_t into k'_{33} .³⁻⁶ This is, however, not always possible, particularly for very high frequency transducers; it may not be even possible to make the height (along the poling direction) larger than the width (perpendicular to the poling direction) for the ceramic constituent in the composite. Also, considering the mechanical stability and the total output power requirement of many transducers, subdicing may not be an option in some situations. Unfortunately, for array elements of arbitrary aspect ratio, there is no known formula available to calculate the electromechanical coupling coefficient. The only thing we can say is that the coupling coefficient value should be in between k'_{33} and k_t , which is obviously not enough.

From a fundamental physics viewpoint, the electromechanical coupling coefficient should be a parameter that can be uniquely determined once the basic material parameters of a piezoelectric material (i.e., the dielectric, piezoelectric, and elastic constants) are given. It is rather confusing to define so many electromechanical coupling coefficients for the same vibration mode along the poling direction. The fundamental problem that leads to those different coupling coefficients is that they were all derived based on certain one-dimensional (1D) approximations without considering mode coupling. Finite element simulations showed that the electromechanical coupling coefficient can be substantially different when the aspect ratio changes.⁷ To address this problem, we

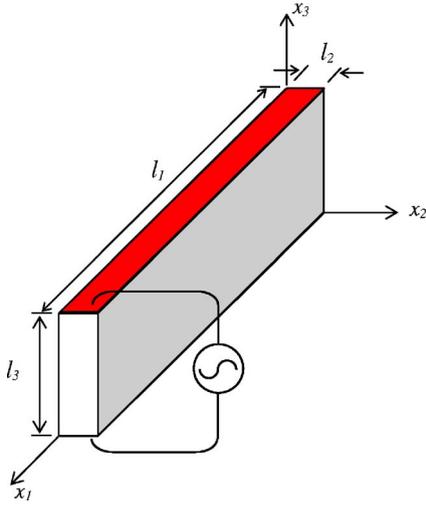


FIG. 1. Illustration of the slender bar piezoelectric resonator used in ultrasonic array transducers.

employ the theory for two-dimensional coupled vibrations and also consider the strain variations with aspect ratio in formulating the unified expression. This unified formula can be used to calculate the electromechanical coupling coefficient for a rectangular slender bar array element of any aspect ratio. Our solution will help clear the conceptual confusion using multiple definitions of electromechanical coupling coefficients as well as provide a convenient and accurate account of the electromechanical coupling coefficient of an array element.

II. THEORY

The dimensions of the array element under study are shown in Fig. 1. The array elements are lined up along the x_2 direction so that the dimension l_2 of each element defines the array pitch, while the dimension l_1 is usually very long so that we can assume a constant strain condition along the x_1 direction. We define the aspect ratio to be $G=l_3/l_2$, where l_3 is the dimension along the poling direction, which determines the resonance frequency of the transducer. Because the x_1 dimension has a constant strain, we only need to consider coupled vibrations along the x_2 and x_3 dimensions.

In general, the resonance frequency of any dimension is inversely proportional to the length of that dimension. Therefore, if l_2 and l_3 differ a lot, the two modes will be well separated with negligible mode coupling. For the case of $l_2 \gg l_3$, or $G \rightarrow 0$, we have a resonant plate (note l_1 is already large) so that the electromechanical coupling coefficient is k_r , while for the case of $l_3 \gg l_2$, or $G \rightarrow \infty$, we have a very tall slender bar resonator, so that the coupling coefficient is k'_{33} . For a more general situation, we must consider coupled vibrations of the two modes along the x_2 and x_3 dimensions.

To avoid complication, we notice that the electrical boundary conditions for this vibrator are $D_1=D_2=E_1=E_2=0$, and there is also a constant strain condition $S_1=0$. In

addition, there are also no shear vibrations involved. Therefore, we only write down the following relevant constitutive relations:

$$T_1 = \sigma T_2 - \frac{s_{13}^E}{s_{11}^E} T_3 - \frac{d_{31}}{s_{11}^E} E_3, \quad (1a)$$

$$S_2 = s_{11}^E (1 - \sigma^2) T_2 + s_{13}^E (1 + \sigma) T_3 + (1 + \sigma) d_{31} E_3, \quad (1b)$$

$$S_3 = s_{13}^E (1 + \sigma) T_2 + \left(s_{33}^E - \frac{(s_{13}^E)^2}{s_{11}^E} \right) T_3 + \left(d_{33} - \frac{s_{13}^E}{s_{11}^E} d_{31} \right) E_3, \quad (1c)$$

$$D_3 = d_{31} (1 + \sigma) T_2 + \left(d_{33} - \frac{s_{13}^E}{s_{11}^E} d_{31} \right) T_3 + \left(\epsilon_{33}^T - \frac{d_{31}^2}{s_{11}^E} \right) E_3, \quad (1d)$$

where $s_{i,j}^E$ ($i, j=1, 3$) are the elastic compliance under constant electric field, $\sigma = -s_{12}^E/s_{11}^E$ is the Poisson's ratio, T_n and S_n ($n=1-6$) are the stress and strain components, E_3 and D_3 are the electric field and electric displacement along x_3 , d_{33} and d_{31} are the piezoelectric constants, and ϵ_{33}^T is the dielectric constant along the poling direction under constant stress.

Considering that the lateral strain $S_2=0$ when $l_2 \gg l_3$ and $S_2 = s_{13}^E (1 + \sigma) T_3 + (1 + \sigma) d_{31} E_3$ when $l_3 \gg l_2$, we can conclude that S_2 must be a function of the aspect ratio G for a general case. We introduce a function $g(G)$, which has the limit of $g(G) \rightarrow 1$ as $G \rightarrow \infty$ and $g(G) \rightarrow 0$ as $G \rightarrow 0$ so that the strain S_2 can be formally written as

$$S_2 = g(G) [s_{13}^E (1 + \sigma) T_3 + (1 + \sigma) d_{31} E_3]. \quad (2)$$

From textbook definition, the internal energy U and the electromechanical coupling coefficient k of a linear piezoelectric resonator are given by⁸

$$U = \frac{1}{2} S_n T_n + \frac{1}{2} D_i E_i = U_e + 2U_m + U_d, \quad \begin{matrix} n = (1-6) \\ i = (1, 2, 3) \end{matrix}, \quad (3)$$

$$k = \frac{U_m}{\sqrt{U_e U_d}}, \quad (4)$$

where, U_e , U_d , and U_m are, respectively, the elastic, dielectric, and coupling energies. Substituting Eqs. (1) and (2) into Eqs. (3) and (4), we have the formal expression of the electromechanical coupling coefficient for the rectangular slender bar array element shown in Fig. 1 with an arbitrary aspect ratio G ,

$$k = \frac{\left[\frac{1+\sigma}{1-\sigma} (g^2(G)-1) \frac{s_{13}^E}{s_{11}^E} d_{31} + \left(d_{33} - \frac{s_{13}^E}{s_{11}^E} d_{31} \right) \right]}{\sqrt{\left[\frac{1+\sigma}{1-\sigma} (g^2(G)-1) \frac{(s_{13}^E)^2}{s_{11}^E} + \left(s_{33}^E - \frac{(s_{13}^E)^2}{s_{11}^E} \right) \right] \left[\frac{1+\sigma}{1-\sigma} (g^2(G)-1) \frac{d_{31}^2}{s_{11}^E} + \left(\varepsilon_{33}^T - \frac{d_{31}^2}{s_{11}^E} \right) \right]}}. \quad (5)$$

In what follows, we will determine the explicit form of $g(G)$.

When the longitudinal and lateral modes (i.e., resonances along the x_3 and x_2 dimensions) are well separated, the $\lambda/2$ resonance frequencies (note: this frequency corresponds to the antiresonant frequency for the longitudinal mode but the resonant frequency for the lateral mode) are given by

$$f_z = \frac{1}{2l_3} \sqrt{\frac{c_{33}^D}{\rho}} = \frac{v_3}{2l_3}, \quad (6a)$$

$$f_y = \frac{1}{2l_2} \sqrt{\frac{c_{11}^E}{\rho}} = \frac{v_2}{2l_2}. \quad (6b)$$

For the small aspect ratio case, i.e., $l_2 \gg l_3$, the resonance frequency f_y is much smaller than f_z considering that the velocities, v_2 and v_3 , are constant and have the same order of magnitude. Therefore, for a small time interval Δt , the displacement ratio ξ_2/ξ_3 along the two dimensions is roughly proportional to the resonance frequency ratio

$$\frac{\xi_2}{\xi_3} \propto \frac{v_2 \Delta t}{v_3 \Delta t} = \frac{l_2 f_y}{l_3 f_z}, \quad (7)$$

or

$$S_2 = \frac{\xi_2}{l_2} \propto \frac{f_y \xi_3}{f_z l_3} = \frac{f_y}{f_z} S_3. \quad (8)$$

The strain S_3 is always finite for all aspect ratios but S_2 strongly depends on the ratio of two resonance frequencies and its value changes from ~ 0 to $s_{13}^E/(1+\sigma)T_3 + (1+\sigma)d_{31}E_3$ as the aspect ratio increases from a very small value to a very large value. Based on this consideration, the frequency ratio should be a factor in the functional form of $g(G)$,

$$g(G) = \kappa(G) \frac{f_1}{f_2} \quad (9)$$

Here $\kappa(G)$ is a function of G to be determined and we use f_1/f_2 in Eq. (9) instead of f_y/f_z to include the mode coupling effect as described below. Obviously, the ratio f_1/f_2 will also depend on the aspect ratio G and it is conceivable that the mode coupling is the strongest when $G \sim 1$, and vanishes as $G \rightarrow 0$ and $G \rightarrow \infty$.

The two-dimensional (2D) coupled elastic vibration problem has been solved by several authors^{9,10} and was also used to calculate the effective electromechanical coupling coefficient of an array element.¹¹ However, the lateral piezoelectric coupling was ignored in Ref. 11 and the solution chosen was a decoupled one. In addition, the integral form of the electromechanical coupling coefficient given in Ref. 11 is

difficult to use in practice. Here, we will derive a convenient universal formula based on a solution including full mode coupling.

First the coupled equations of motion for the local displacement \mathbf{u} along x_2 and x_3 directions are given by

$$\rho \frac{\partial^2 u_2}{\partial t^2} = c_{11}^E \frac{\partial^2 u_2}{\partial x_2^2} + c_{13}^E \frac{\partial^2 u_2}{\partial x_2 \partial x_3}, \quad (10a)$$

$$\rho \frac{\partial^2 u_3}{\partial t^2} = c_{13}^D \frac{\partial^2 u_3}{\partial x_2 \partial x_3} + c_{33}^D \frac{\partial^2 u_3}{\partial x_3^2}, \quad (10b)$$

where

$$c_{13}^D = c_{13}^E + \frac{e_{31}e_{33}}{\varepsilon_{33}^S}, \quad c_{33}^D = c_{33}^E + \frac{(e_{33})^2}{\varepsilon_{33}^S}. \quad (10c)$$

Note that the elastic constants are constant D values for vibration along the poling direction but constant E values for vibrations perpendicular to the poling direction; ε_{33}^S is the dielectric constant under constant strain.

Considering the geometry and the conservation of momentum, we chose the following solutions for the coupled equations:

$$u_2 = A_2 \sin k_2 x_2 \cos k_3 x_3 \cos \omega t, \quad (11a)$$

$$u_3 = A_3 \cos k_2 x_2 \sin k_3 x_3 \cos \omega t. \quad (11b)$$

Substituting Eqs. (11a) and (11b) into Eqs. (10a) and (10b) leads to an eigenvalue problem,

$$\begin{vmatrix} (\omega^2 \rho - \omega_1^2 \rho) & -c_{13}^E \frac{\rho}{\sqrt{c_{11}^E c_{33}^D}} \omega_1 \omega_3 \\ -c_{13}^D \frac{\rho}{\sqrt{c_{11}^E c_{33}^D}} \omega_1 \omega_3 & (\omega^2 \rho - \omega_3^2 \rho) \end{vmatrix} = 0, \quad (12)$$

where $\omega_1^2 = c_{11}^E k_1^2 / \rho$, $\omega_3^2 = c_{33}^D k_3^2 / \rho$.

Define $\omega_1 = 2\pi f_y$ and $\omega_3 = 2\pi f'_z$, then Eq. (12) becomes

$$[(f)^2 - (f'_z)^2][(f)^2 - (f_y)^2] = \Gamma^2 (f'_z)^2 (f_y)^2, \quad (13)$$

where Γ is the mode coupling constant given by

$$\Gamma = \frac{c_{13}^E}{\sqrt{c_{33}^E c_{11}^E}}. \quad (14)$$

In Eq. (13), the frequencies f'_z and f_y , are the resonant frequencies along the x_3 and x_2 dimensions without mode coupling, respectively. The expression for f_y is given in Eq. (6b) while the expressions for the longitudinal resonance f'_z is given by

$$f'_z = \frac{1}{2l_3} \frac{2X_t}{\pi} \sqrt{\frac{c_{33}^D}{\rho}}, \tag{15}$$

where X_t is the first root of the transcendental equation

$$1 - k_t^2 \frac{\tan X}{X} = 0. \tag{16}$$

Near the resonance, the solution may be approximated as

$$X_t \approx \frac{1}{2} \pi \left(1 - \frac{4k_t^2}{\pi^2} \right). \tag{17}$$

Solving Eq. (13) leads to the following solutions:

$$l_3 f_1 = \sqrt{\frac{G^2 c_{11}^E}{8\rho} + \frac{c_{33}^D X_t^2}{2\pi^2 \rho} - \frac{\sqrt{-16\pi^2 c_{11}^E c_{33}^D X_t^2 (1-\Gamma^2) G^2 + (c_{11}^E \pi^2 G^2 + 4c_{33}^D X_t^2)^2}}{8\pi^2 \rho}}, \tag{18a}$$

$$l_3 f_2 = \sqrt{\frac{G^2 c_{11}^E}{8\rho} + \frac{c_{33}^D X_t^2}{2\pi^2 \rho} + \frac{\sqrt{-16\pi^2 c_{11}^E c_{33}^D X_t^2 (1-\Gamma^2) G^2 + (G^2 c_{11}^E \pi^2 + 4c_{33}^D X_t^2)^2}}{8\pi^2 \rho}}. \tag{18b}$$

Note that the frequency f_1 is always lower than f_2 as shown in Fig. 2, hence, f_1 always represents the resonant frequency of the larger dimension. In other words, for the aspect ratio smaller than 1, f_1 represents the vibration frequency of the lateral mode along x_2 , while for aspect ratio larger than 1, f_1 represents the vibration frequency of the longitudinal mode along x_3 .

In order to determine the form of $\kappa(G)$ in Eq. (9) we take the limit of $g(G)$ as $G \rightarrow \infty$ using the solutions of Eq. (18). As mentioned above, for a very large aspect ratio, we want the limit of $g(G)$ to be 1, i.e.,

$$\lim_{G \rightarrow \infty} \kappa(G) \frac{f_1}{f_2} = \lim_{G \rightarrow \infty} \frac{2\kappa(G)}{G\pi} \sqrt{\frac{c_{33}^D}{c_{11}^E} (1-\Gamma^2) X_t} = 1. \tag{19}$$

This requires $\kappa(G)$ to have the following form:

$$\kappa(G) = \frac{G}{2} \frac{\pi}{X_t} \sqrt{\frac{c_{11}^E}{c_{33}^D (1-\Gamma^2)}}. \tag{20}$$

Therefore, the function $g(G)$ is given by

$$g(G) = \frac{G}{2} \frac{\pi}{X_t} \sqrt{\frac{c_{11}^E}{c_{33}^D (1-\Gamma^2)}} \frac{f_1}{f_2}. \tag{21}$$

It can be easily verified that $g(G) \rightarrow 0$ as $G \rightarrow 0$ because the limit of $f_1/f_2 \rightarrow 0$ as $G \rightarrow 0$. Substituting Eq. (21) into Eq. (5) gives us the explicit universal formula for the electromechanical coupling coefficient k as a function of the aspect ratio G .

We now test the end limits of the formula. When $G \rightarrow 0$, $g(G) \rightarrow 0$, Eq. (5) recovers the expression of k_t , which is the electromechanical coupling coefficient for a thin plate,

$$\begin{aligned} \lim_{G \rightarrow 0} k &= \frac{d_{33} - \frac{2d_{31}s_{13}^E}{(s_{11}^E + s_{12}^E)}}{\sqrt{\left(s_{33}^E - \frac{2(s_{13}^E)^2}{(s_{11}^E + s_{12}^E)}\right) \left(\epsilon_{33}^T - \frac{2d_{31}^2}{(s_{11}^E + s_{12}^E)}\right)}} \\ &= \sqrt{\frac{e_{33}^2}{c_{33}^D \epsilon_{33}^S}} = k_t. \end{aligned} \tag{22}$$

In Eq. (22) we have used the relationships $s_{33}^E - 2(s_{13}^E)^2/s_{11}^E + s_{12}^E = 1/c_{33}^E$ and $c_{33}^D = c_{33}^E + e_{33}^2/\epsilon_{33}^S$ between materials constants obtained at different boundary conditions in order to convert the limit into a standard form of k_t .

When $G \rightarrow \infty$, $g(G) \rightarrow 1$, Eq. (5) becomes the coupling coefficient of k'_{33} , which is the electromechanical coupling coefficient for a very tall rectangular slender bar resonator,

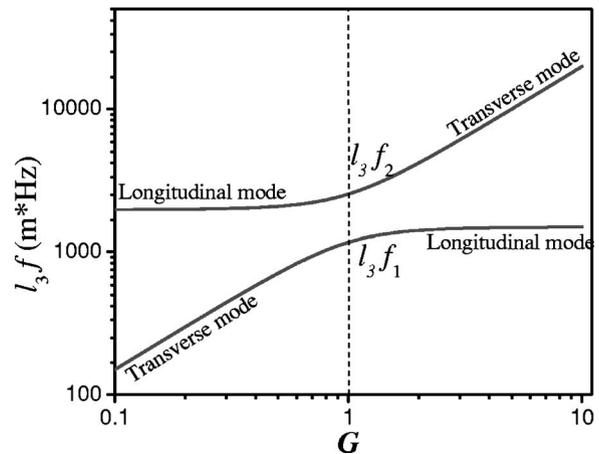


FIG. 2. Frequency constant as a function of the aspect ratio G for a slender bar resonator showing in Fig. 1 based on mode coupling theory.

TABLE I. Material parameters of PZT-5 ceramic.^a

| Elastic properties $c_{\lambda\mu}$ (N/m ²), $s_{\lambda\mu}$ (m ² /N) | Piezoelectric constant $e_{i\lambda}$ (C/m ²), $d_{i\lambda}$ (C/N) | Dielectric constant | Density (kg/m ³) |
|--|--|--------------------------|---------------------------------|
| $c_{11}^E = 121 \times 10^9$ | $e_{31} = -5.4$ | $\epsilon_{33}^S = 830$ | $\rho = 7750$ |
| $c_{12}^E = 75.4 \times 10^9$ | $e_{33} = 15.8$ | $\epsilon_{33}^T = 1700$ | |
| $c_{13}^E = 75.2 \times 10^9$ | | | |
| $c_{33}^E = 111 \times 10^9$ | | | |
| $s_{11}^E = 16.4 \times 10^{-12}$ | $d_{31} = -171 \times 10^{-12}$ | | |
| $s_{12}^E = -5.74 \times 10^{-12}$ | $d_{33} = 373 \times 10^{-12}$ | | |
| $s_{13}^E = -7.22 \times 10^{-12}$ | | | |
| $s_{33}^E = 18.8 \times 10^{-12}$ | | | |

^aThese data are from D. Berlincourt *et al.*, see Ref. 8.

$$\lim_{G \rightarrow \infty} k = \frac{d_{33} - \frac{s_{13}^E}{s_{11}^E} d_{31}}{\sqrt{\left(s_{33}^E - \frac{(s_{13}^E)^2}{s_{11}^E}\right) \left(\epsilon_{33}^T - \frac{d_{31}^2}{s_{11}^E}\right)}} = k'_{33}. \quad (23)$$

III. NUMERICAL APPLICATIONS AND DISCUSSIONS

Using the derived general formula Eq. (5) with the function $g(G)$ given in Eq. (21), we have calculated the aspect ratio dependence of the electromechanical coupling coefficient k of an array element with arbitrary aspect ratio using the material properties of PZT-5 and BaTiO₃ ceramics. The material parameters taken from Ref. 8 are given in Tables I and II.

Figure 3 shows the general behavior of the electromechanical coupling coefficient of a rectangular slender bar array element as a function of the aspect ratio. It is a kink-type solution and varies quickly in the vicinity of $G=1$. Practically speaking, severe variation of k with aspect ratio happens when the aspect ratio is between $G=1/3$ and $G=3$. For convenience of discussion, we list some k values for a few typical aspect ratios in Table III. One can see that for an aspect ratio of $G=3$, the k value is already reaching 98.4% of the k'_{33} value while for the case of aspect ratio of $G=1/4$, the k value is practically the same as k_t . Therefore, for better performance, the aspect ratio of the element should be larger than 3.

TABLE II. Material parameters of BaTiO₃ ceramic.^a

| Elastic properties $c_{\lambda\mu}$ (N/m ²), $s_{\lambda\mu}$ (m ² /N) | Piezoelectric constant $e_{i\lambda}$ (C/m ²), $d_{i\lambda}$ (C/N) | Dielectric constant | Density (kg/m ³) |
|--|--|--------------------------|---------------------------------|
| $c_{11}^E = 150 \times 10^9$ | $e_{31} = -4.35$ | $\epsilon_{33}^S = 1260$ | $\rho = 5700$ |
| $c_{12}^E = 66 \times 10^9$ | $e_{33} = 17.5$ | $\epsilon_{33}^T = 1700$ | |
| $c_{13}^E = 66 \times 10^9$ | | | |
| $c_{33}^E = 146 \times 10^9$ | | | |
| $s_{11}^E = 9.1 \times 10^{-12}$ | $d_{31} = -78 \times 10^{-12}$ | | |
| $s_{12}^E = -2.7 \times 10^{-12}$ | $d_{33} = 190 \times 10^{-12}$ | | |
| $s_{13}^E = -2.9 \times 10^{-12}$ | | | |
| $s_{33}^E = 9.5 \times 10^{-12}$ | | | |

^aThese data are from D. Berlincourt *et al.*, see Ref. 8.

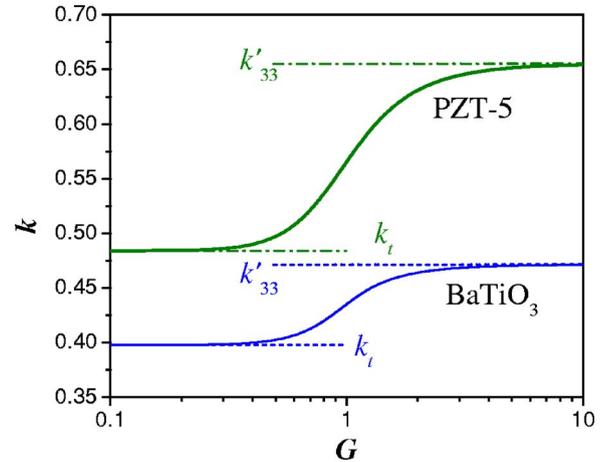


FIG. 3. Electromechanical coupling coefficient of a rectangular slender bar resonator as a function of the aspect ratio. The piezoelectric materials used are PZT-5 and BaTiO₃ ceramics.

IV. SUMMARY AND CONCLUSIONS

For a piezoelectric vibrator operating along the poling direction, there are three different electromechanical coupling coefficients defined in the literature for extreme geometries in which 1D solutions can be obtained. They are k_{33} for a long rod, k'_{33} for a very tall slender bar, and k_t for a thin plate.² These definitions are not only confusing in basic concept (all for the same vibration mode) but also very limited in describing practical situations since they cannot describe resonators that do not have such extreme geometries. The main problem in previous treatments is that the mode coupling effect was ignored and no aspect ratio effect was considered. For the rectangular slender bar vibrator, such as the element in phase and linear ultrasonic transducer arrays shown in Fig. 1, the problem must be treated as 2D coupled vibrations. The effect of coupling between the two modes along the two dimensions is a function of the aspect ratio. It is the strongest when $G \sim 1$ and vanishes as $G \rightarrow \infty$ and as $G \rightarrow 0$.

Based on the theory of coupled vibrations and the changing trend of the lateral strain, we have derived a unified formula for the electromechanical coupling coefficient of a rectangular slender bar transducer array element. Our formula can provide accurate description of the electromechanical coupling coefficient for resonators that do not satisfy the aspect ratio requirements for k'_{33} or k_t mode and will converge to k'_{33} and k_t for large and small aspect ratios, respectively. Based on our results, there is no need to define so many different coupling coefficients for the same mode. This not only makes the physical concept much clearer than be-

TABLE III. Electromechanical coupling coefficient for a few selected aspect ratios.

| | G | 0 | 1/4 | 1/2 | 1 | 2 | 3 | 4 | ∞ |
|--------------------|---------|-------|-------|-------|-------|-------|-------|-------|-------------|
| PZT | $k(G)$ | 0.484 | 0.485 | 0.497 | 0.566 | 0.631 | 0.645 | 0.649 | 0.655 |
| | (k_t) | | | | | | | | (k'_{33}) |
| BaTiO ₃ | $k(G)$ | 0.398 | 0.398 | 0.402 | 0.435 | 0.464 | 0.468 | 0.470 | 0.471 |
| | (k_t) | | | | | | | | (k'_{33}) |

fore, but also provides a useful formula for transducer engineers to design array elements that have arbitrary aspect ratios.

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