

## Experimental technique for characterizing arbitrary aspect ratio piezoelectric resonators

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(Received 23 August 2006; accepted 11 September 2006; published online 20 October 2006)

The electromechanical coupling coefficient  $k$  is the most important parameter for the characterization of a piezoelectric material and for the design of electromechanical devices. Standard resonance technique for measuring the  $k$  value is based on one-dimensional approximation, which needs to use samples having extreme geometries. For small size crystals and/or due to geometrical constraints in many devices, piezoelectric resonators may not be made into such extreme geometries. An averaging scheme has been developed to tackle this challenging experimental task, and the so obtained  $k$  values agree well with theoretical predictions of  $k$  values for arbitrary aspect ratio resonators. © 2006 American Institute of Physics.

[DOI: 10.1063/1.2364465]

For the characterization of a piezoelectric material and for the design of electromechanical devices, it is important to know the electromechanical coupling coefficient  $k$ , which reflects the electromechanical coupling strength and is defined by  $k = U_m / \sqrt{U_e U_d}$ , where  $U_m$  is the electromechanical coupling energy and  $U_e$  and  $U_d$  are the elastic and dielectric energies, respectively.<sup>1</sup> However, the electromechanical coupling strength is strongly dependent on the aspect ratio of the piezoelectric resonator; hence, there are different  $k$  values defined for the same type of vibration. For example, for the vibration along the poling direction, there are  $k_{33}$ ,  $k'_{33}$ , and  $k_t$  defined corresponding to a long thin bar, a tall rectangular slab, and a thin plate, respectively. The physical origin of this aspect ratio dependence comes from the fact that the electromechanical energy conversion is strongly influenced by boundary conditions as well as by mode coupling effect when the frequencies of two or more modes are coupled to each other. As can be found in the literature, the difference of the coupling coefficient values for different aspect ratios can be very large. For example, in the case of  $\text{Pb}(\text{Zr},\text{Ti})\text{O}_3$  (PZT) ceramic resonators,  $k_{33}$  is as high as 70%, while  $k'_{33}$  is about 65% and  $k_t$  is only about 48%. All three  $k$  values can be written in terms of material constants, and the corresponding formulas are derived from different types of one-dimensional (1D) approximations. These multiple  $k$  values are not only inconvenient but also insufficient when it comes to describing a resonator that does not have such extreme aspect ratios.

Recently, based on mode coupling theory and the fundamental energy ratio definition of the electromechanical coupling coefficient, a unified formula has been derived<sup>2</sup> that can be used to calculate the electromechanical coupling coefficient for any given aspect ratio of a rectangular bar resonator, such as those used in ultrasonic array transducers (Fig. 1). The unified formula is given by<sup>2</sup>

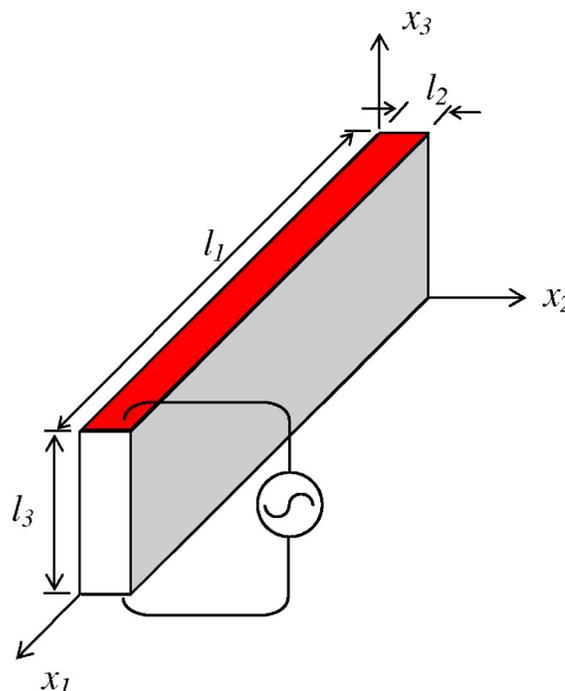


FIG. 1. (Color online) Illustration of the resonator and its dimensions.

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$$k = \frac{[[ (1 + \sigma)/(1 - \sigma) ] (g^2(G) - 1) (s_{13}^E/s_{11}^E) d_{31} + (d_{33} - (s_{13}^E/s_{11}^E) d_{31})]}{\sqrt{[[ (1 + \sigma)/(1 - \sigma) ] (g^2(G) - 1) [(s_{13}^E)^2/s_{11}^E] + (s_{33}^E - (s_{13}^E)^2/s_{11}^E) ] [[ (1 + \sigma)/(1 - \sigma) ] (g^2(G) - 1) (d_{31}^2/s_{11}^E) + (\epsilon_{33}^T - d_{31}^2/s_{11}^E) ]}}, \tag{1}$$

where the aspect ratio is defined as  $G=l_3/l_2$  and the dimension  $l_1$  is kept long so that the boundary condition along  $x_1$  can be treated as a constant strain condition, which reduces the three-dimensional coupling problem to a two-dimensional one,  $s_{ij}^E$  are components of the elastic compliance,  $d_{ij}$  are the piezoelectric coefficients, and the function  $g(G)$  is given by<sup>2</sup>

$$g(G) = \frac{G \pi}{2 X_t} \sqrt{\frac{c_{11}^E}{c_{33}^D (1 - \Gamma^2)}} \frac{f_1}{f_2}. \tag{2}$$

The two frequencies are defined by

$$l_3 f_1 = \sqrt{\frac{G^2 c_{11}^E}{8\rho} + \frac{c_{33}^D X_t^2}{2\pi^2 \rho} - \frac{\sqrt{-16\pi^2 c_{11}^E c_{33}^D X_t^2 (1 - \Gamma^2) G^2 + (c_{11}^E \pi^2 G^2 + 4c_{33}^D X_t^2)^2}}{8\pi^2 \rho}}, \tag{3a}$$

$$l_3 f_2 = \sqrt{\frac{G^2 c_{11}^E}{8\rho} + \frac{c_{33}^D X_t^2}{2\pi^2 \rho} + \frac{\sqrt{-16\pi^2 c_{11}^E c_{33}^D X_t^2 (1 - \Gamma^2) G^2 + (G^2 c_{11}^E \pi^2 + 4c_{33}^D X_t^2)^2}}{8\pi^2 \rho}}, \tag{3b}$$

where  $\Gamma = c_{13}^E / \sqrt{c_{33}^E c_{11}^E}$  and  $X_t$  is the first root of the transcendental equation

$$1 - k_t^2 \frac{\tan X}{X} = 0. \tag{4}$$

As expected, Eq. (1) is a kink-type monotonic function of  $G$  and changes from  $k_t$  to  $k'_{33}$  as  $G$  varies from a very small value to a very large value.

Experimentally, the  $k$  values can be obtained from the electrical impedance spectrum based on the 1D formula<sup>1,3</sup>

$$k = \sqrt{\left( \frac{\pi f_r}{2 f_a} \right) \cot \left( \frac{\pi f_r}{2 f_a} \right)}, \tag{5}$$

where  $f_r$  and  $f_a$  are the resonance and antiresonance frequencies, respectively. However, we cannot use Eq. (5) for reso-

nators with arbitrary aspect ratios because the formula is good only for the case of very large  $G$  and very small  $G$  cases. In addition, the resonance and antiresonance frequencies cannot be determined correctly when there are mode couplings. In order to resolve this problem, we adopt an averaging scheme to deal with the mode coupling effect as detailed below.

Based on the equivalent circuit theory for a piezoelectric resonator, near a resonance the piezoelectric resonator can be modeled by an *LRC* resonance circuit, as shown in Fig. 2(a), and the effective electromechanical coupling coefficient  $k_{\text{eff}}$  may be calculated from the following equation:<sup>4,5</sup>

$$k_{\text{eff}}^2 = \frac{C^T - C_0}{C^T} = 1 - \frac{C_0}{C^T}, \tag{6}$$

where  $C_0$  is the clamped capacitance,  $C^T = C_0 + C_1$  is the low frequency limit of the capacitance, and  $C_1$  is the capacitance in series with the effective inductance. Without mode coupling, the frequency spectrum of the susceptance (the imaginary part of the admittance) is illustrated in Fig. 2(b), and the  $C_0$  and  $C^T$  values can be derived from the slopes of the two asymptotic straight lines  $\omega C_0$  (dotted line) and  $\omega C^T$  (dashed line) derived using data far above and far below the resonance and antiresonance frequencies. When mode coupling occurs, multiple peaks will appear in the electrical impedance and the peak positions will be shifted; however, we found that the average slope of the susceptance spectra does not change significantly. Based on this fact, we propose to use the average slopes of the susceptance spectra to estimate the  $C_0$  and  $C^T$  values when there are mode couplings. Then, use them to calculate the electromechanical coupling coefficient.

To verify this idea, we made seven resonators with different aspect ratios using a Motorola PZT3203HD ceramic. In order to minimize sample to sample variation caused by the fabrication process, only  $l_2$  is different for different resonators, while the other two dimensions are kept the same:  $l_1 = 10$  mm and  $l_3 = 1$  mm. These samples were all cut from the same uniform plate to guarantee data consistency. After cutting and polishing, the samples were repoled to restore

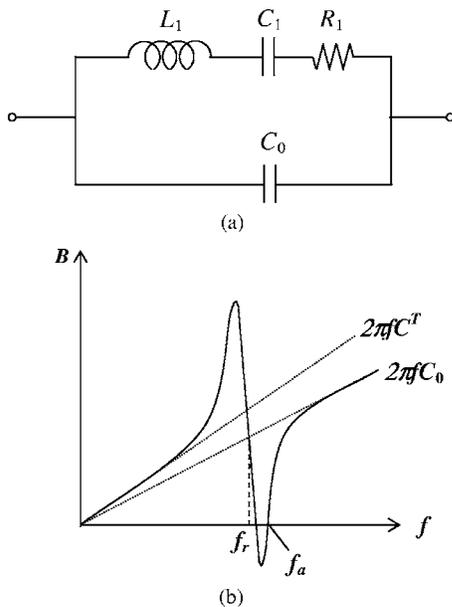


FIG. 2. (a) Equivalent circuit of a piezoelectric resonator near a resonance. (b) Capacitances  $C^T$  and  $C_0$  are related to the slopes of the linear part before and after the resonance in the frequency spectrum of the susceptance.

TABLE I. Measured material constants of Motorola PZT3203HD.

| Elastic properties<br>$c_{\lambda\mu}$ (N/m <sup>2</sup> ), $s_{\lambda\mu}$ (m <sup>2</sup> /N) | Piezoelectric constant<br>$e_{i\lambda}$ (C/m <sup>2</sup> ), $d_{i\lambda}$ (C/N) | Dielectric constant       | Density<br>(kg/m <sup>3</sup> ) |
|--|--|---------------------------|---------------------------------|
| $c_{11}^E=141.3 \times 10^9$   | $e_{31}=-3.69$   | $\epsilon_{33}^S=1461.93$ | $\rho=7813.7$                   |
| $c_{12}^E=84.8 \times 10^9$  | $e_{33}=26.84$   | $\epsilon_{33}^T=3876.33$ |                                 |
| $c_{13}^E=95.0 \times 10^9$  |  |                           |                                 |
| $c_{33}^E=122.0 \times 10^9$   |  |                           |                                 |
| $s_{11}^E=15.25 \times 10^{-12}$   | $d_{31}=-314.62 \times 10^{-12}$   |                           |                                 |
| $s_{12}^E=-2.45 \times 10^{-12}$   | $d_{33}=710.00 \times 10^{-12}$  |                           |                                 |
| $s_{13}^E=-9.96 \times 10^{-12}$   |  |                           |                                 |
| $s_{33}^E=23.71 \times 10^{-12}$   |  |                           |                                 |

their optimum piezoelectric properties. Using ultrasonic and resonance techniques,<sup>6-8</sup> we have measured the full matrix properties of the PZT3203HD ceramic, and the data are listed in Table I. For consistency check, four identical resonators were prepared for each specified aspect ratio. Direct material property characterization showed that the property variation from sample to sample is less than 0.2% after the repoling procedure.

The admittance spectra of these samples are measured using an HP4294A precision impedance analyzer. Typical susceptance spectrum is shown in Fig. 3. The averaging straight lines used for estimating  $C_0$  and  $C^T$  values are also shown in the figure. For the  $G=1$  case, strong mode coupling occurs, as can be seen from Fig. 3. Such mode coupling invalidates the 1D formula [Eq. (5)] because all resonance

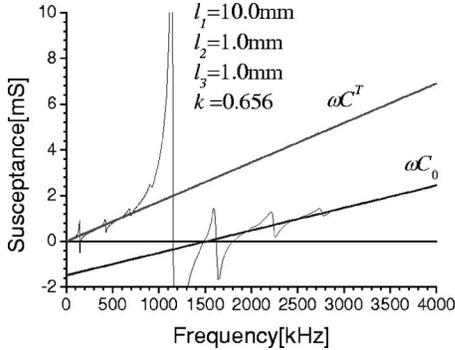


FIG. 3. Typical measured susceptance spectra for  $G=1$ .

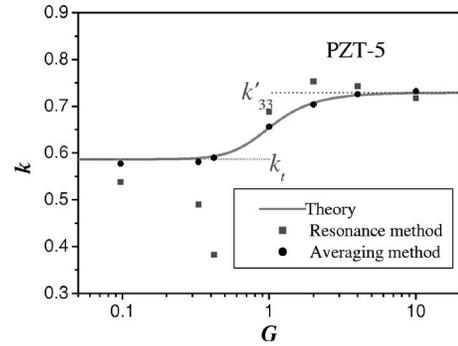


FIG. 4. Electromechanical coupling coefficient as a function of the aspect ratio. The line is from the unified formula [Eq. (1)], the squares are results calculated using Eq. (5), and the solid circles are results from the averaging scheme.

frequencies are being shifted due to mode coupling.

Based on the slopes of these susceptance spectra, the electromechanical coupling coefficient for each case is calculated using Eq. (6), and the results are shown in Fig. 4 together with the theoretically predicted curve from Eq. (1) and the  $k$  values calculated using the classical resonance 1D formula [Eq. (5)]. Obviously, the 1D formula does not work when there is a mode coupling. On the other hand, the measured effective  $k$  values using the averaging slope scheme are in excellent agreement with the theoretical predictions. Hence, this averaging scheme provides an experimental method to estimate the electromechanical coupling coefficient of a resonator with an arbitrary aspect ratio.

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