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ELASTIC PROPERTY CHARACTERIZATION IN THIN SAMPLES OF SUB-WAVELENGTH IN THICKNESS

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A conflict exists in ultrasonic measurements between the resolution which requires higher frequency, and the penetration depth which requires long wavelength. Traditional pulse-echo method for elastic property measurements fails when the sample becomes too thin to allow the separation of repeated echoes. A data processing technique is described here which may provide a solution to this conflict. Elastic properties were successfully measured in samples as thin as 5% of the wavelength λ.

Keywords: Ultrasonic; thin layers; sub-wavelength; high resolution; penetration depth

INTRODUCTION

One of the most powerful techniques for elastic property characterization is ultrasonic method. The pulse-echo technique[1–4] has been well developed and widely used in characterizing solid materials. Sound velocity, which directly reflect the elastic property, can be measured through the time delay between consecutive echoes. Ultrasound is also a major tool in medical diagnosis in which the elastic properties are known and the pulse-echo pattern is used to form the imaging of an object inside a body. Because pulse-echo technique is to send a tone-burst ultrasonic signal into the structure and measure the time lagging between consecutive echoes, its resolution is limited by the pulse-width. Using a good broadband short ringdown transducer, one can achieve an axial resolution of $1-2\lambda$, where λ is the wavelength. Both transmission and reflection mode operations in ultrasonic measurements will produce similar echo patterns but different amplitudes depending on the acoustic impedance matching between the medium and the sample. A pair of parallel surfaces of the sample will serve as the reflection planes so that a series of echoes will be generated as shown in Figure 1. The time delay between the adjacent echoes is the time for an acoustic pulse to go through the sample one round trip.

It is hard to achieve sub-wavelength resolution as one can imagine from Figure 1. The pulse separation reflects the sample thickness. If the sample is too thin, these consecutive echoes will overlap to produce a global profile. As demonstrated in Figure 2, when the delay time of two consecutive echoes is less than the ringdown time of the transducer, they cannot be distinguished. The overlapping problem becomes worse for materials with low attenuation because the third echo, the fourth echo, and so on, will all contribute to the measured echo profile, making the measurement impossible.

Although the resolution of ultrasonic technique can be increased through increasing operating frequency, the attenuation increases even faster with frequency. Generally speaking, the attenuation is proportional to f^{β}, with $1 < \beta < 2$ for ordinary materials and $\beta > 2$ for lossy materials. In other words, higher frequency ultrasonic waves would have much shallower penetration depth, which makes the detection of bonding layer or any imbedded thin object almost impossible using ultrasonic technique, because low frequency waves do not see it while high frequency signals cannot penetrate the structure.

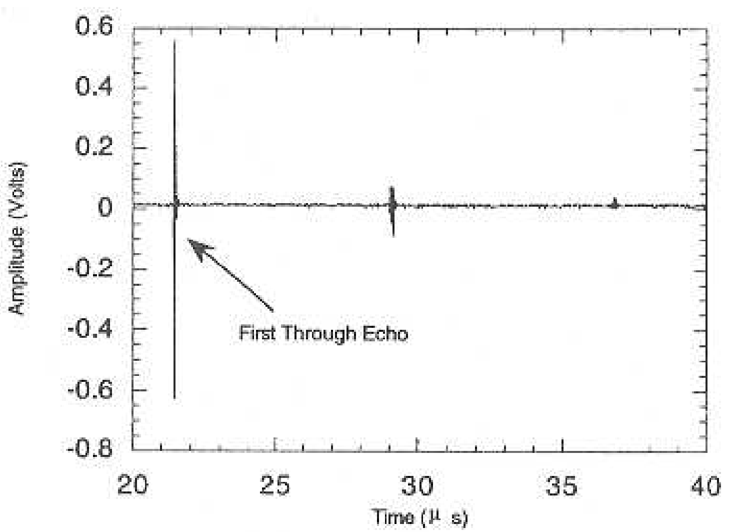

FIGURE 1 The echoes caused by the repeated reflections inside the sample. The signal is obtained from a through mode operation using a 2 MHz center frequency broadband transducer.

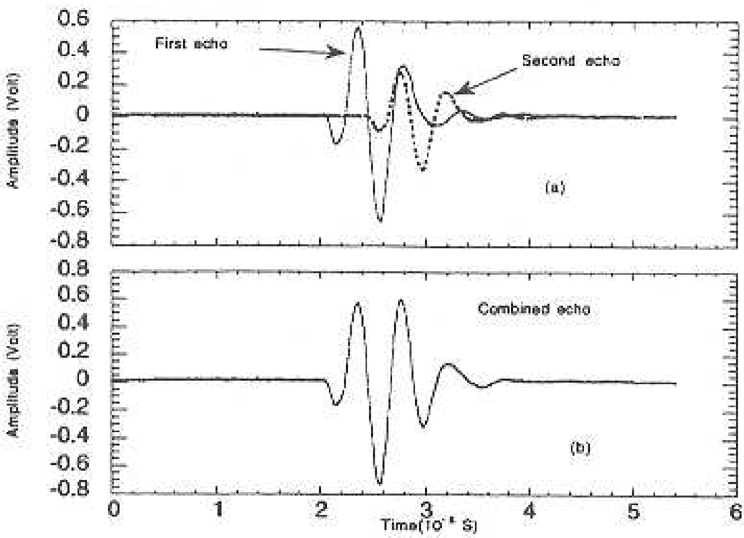

FIGURE 2 Echo overlapping in a thin sample. (a) The two consecutive echoes if not interfere with each other. (b) The actually observed overlapped signal.

The challenge is how to achieve high resolution and deep penetration at the same time. Obviously, the only way to address this problem is to use low frequency signal and achieve sub-wavelength resolution. A method is described in this paper which can be used to measure samples as thin as 5% of λ.

DATA PROCESSING SCHEME

For a thin sample, the received signal from transmission ultrasonic measurement will be a mixture of consecutive transmitted signals due to the multiple reflections within the sample. If we do not consider the attenuation of the sample, the received signal, $x(r, t)$, can be written in terms of the input signal function $s(r, t)$ in the following form:

$$x(r, t) = T_{21} T_{12} \sum_{i=0}^{\infty} R_{21}^{2i} s\left(t - \frac{(r-l)}{V_m} - \frac{(2i+1)l}{V_s}\right) \tag{1}$$

where T_{21} and T_{12} are the transmission coefficients from the medium to the sample and from the sample to the medium, respectively, R_{21} is the reflection coefficient from sample to the medium, l is the thickness of the sample, V_m and V_s are the sound velocities of the medium and the sample, respectively.

Although the multi-reflections within the sample are not desired in the traditional ultrasonic measurement, such multi-reflections do actually carry strong signature of the sample. One can obtain the desired information using more elaborate signal processing schemes.

A simple and direct way to abstract the time of flight inside the sample is to find the response function, $h(t)$, so that the transmitted singal $x(t)$ becomes the convolution of the input singal $s(t)$ and $h(t)$,

$$x(t) = \mathrm{conv}\,[\,h(t), s(t)\,] = \int_{-\infty}^{\infty} h(\tau)s(t-\tau)d\tau \tag{2}$$

For simplicity, we have shifted the time reference and set the space variable to be zero at the exit surface of the sample.

For an ideal case, the response function of the sample is simply a series of δ-functions:

$$h(t) = \sum_{j=0}^{\infty} A^j \delta(t - \Delta t), \tag{3}$$

where

$$A = T_{21}\, T_{12}\, R_{12}, \quad \Delta t = \frac{2\,l}{V_s}. \tag{4}$$

In principle, if one can derive the response function $h(t)$ from the measured signals, the round trip time Δt for wave to travel inside the sample can be easily obtained from the peak separation of the δ-function series. Unfortunately, there is no easy way to get "clean" data in practical measurements. All the signals are convoluted by instrumental response functions and being added to certain level of noises, both acoustically and electronically.

In transmission mode operation, the input signal $s(t)$ and the transmitted signal $x(t)$ can be obtained by removing and placing the sample in the acoustic pathway, respectively. The experiementally observed signals (quantities with an over-bar) are not clean and may be written in the following forms:

a) Observed input signal:

$$\overline{s(t)} = \mathrm{conv}\,[\,g(t), s(t)\,] + n_s(t) \tag{5}$$

b) Observed transmitted signal:

$$\overline{x(t)} = \mathrm{conv}[\,g(t), \mathrm{conv}\,[\,h(t), s(t)\,]] + n_x(t) \tag{6}$$

where $g(t)$ is the response function of the experimental set-up, $n_s(t)$ and $n_x(t)$ represent the noises that are being added to the true signals.

In order to develop a universal method, we like to avoid the task of having to characterize each instrument for its response function $g(t)$. This can be done by defining a new function $k(t)$ to be the modified signal input,

$$k(t) = \text{conv}\,[g(t), s(t)], \tag{7}$$

and utilizing the property of convolution integral:

$$\text{conv}\,[g(t), \text{conv}\,[h(t), s(t)]] = \text{conv}[h(t), \text{conv}\,[g(t), s(t)]] \tag{8}$$

Hence, the input and transmitted signals become

$$\overline{s(t)} = k(t) + n_s(t) \tag{9}$$

$$\overline{x(t)} = \text{conv}\,[h(t), k(t)] + n_x(t) \tag{10}$$

The response function can be formally written in the frequency domain as[5]:

$$H(f) = \frac{\overline{X(f)}}{K(f)}\,\phi(f) = \frac{\overline{X(f)}K^*(f)}{|K(f)|^2}\,\phi(f) \tag{11}$$

where $\phi(f)$ is a Weiner optimal filtering factor [5] which can be written in terms of the power spectra of $\overline{x(t)}$ and $n_x(t)$.

In order to derive the power spectrum of the noise $n_x(t)$, we can perform multiple sampling, i.e., taking the average of m_1 times measurements of the transmitted signal $\langle\overline{x(t)}\rangle_{m_1}$ and then taking another average of m_2 times measurements $\langle\overline{x(t)}\rangle_{m_2}$, with $m_2 > m_1$. It can be shown that the noise power spectrum may be replaced by these averages and the Weiner optimal filtering factor $\phi(f)$ can be written as the following[6],

$$\phi(f) = 1 - \frac{m_1 m_2}{(m_1 - m_2)|\overline{X(f)}|^2}\left(|\overline{X_{m1}(f)}|^2 - |\overline{X_{m2}(f)}|^2\right) \tag{12}$$

From Eqs. (11) and (12), the spectrum of the sample response function $h(t)$ is given by

$$H(f) = \frac{\overline{X(f)}\,\overline{S^*(f)}}{|K(f)|^2}\,\phi(f) \tag{13}$$

Then the response function of the specimen, $h(t)$, can be obtained from $H(f)$ using inverse Fourier Transform. We have implemented the FFT algorithm in the data processing program so that the analysis is almost real time.

RESULTS

This signal processing scheme described above has been applied to a set of brass shims purchased from Precision Brand, Downers Grove, Illinois. The thickness of the shims ranges from 0.004″ – 0.025″. Figures 3 and 4 show the measurement results on a 0.004″ and a 0.008″ thick brass shims, respectively. Figures 3(a) and 4(a) are the input and transmitted signals, which were

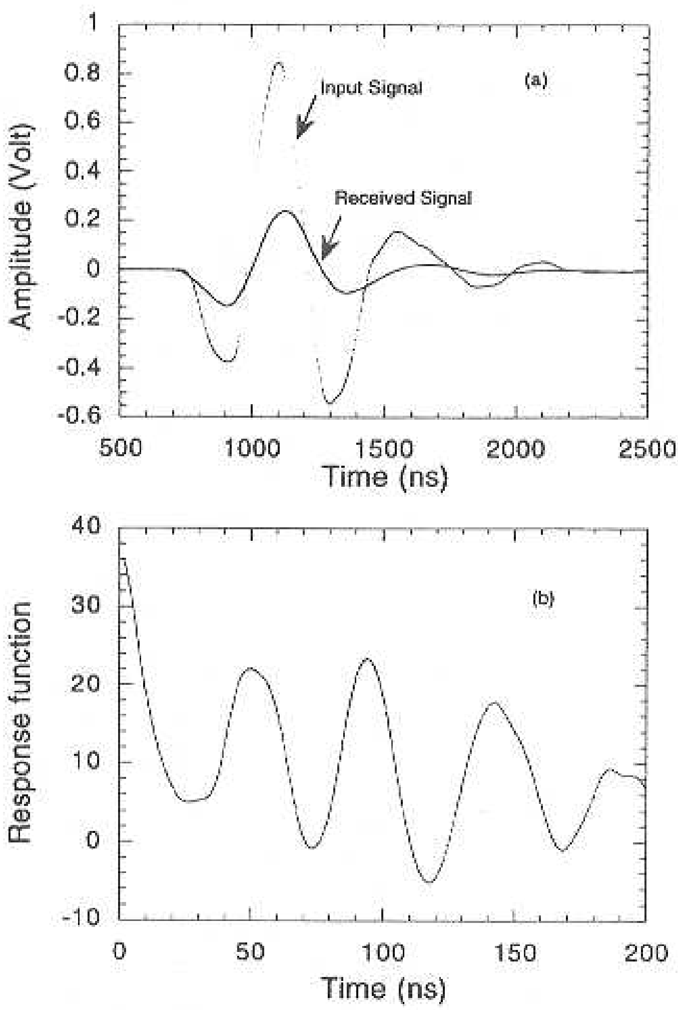

FIGURE 3 (a) The input signal to the sample and the received signal after the pulse went through the sample and (b) The response function derived from (a). The sample thickness is 0.004″.

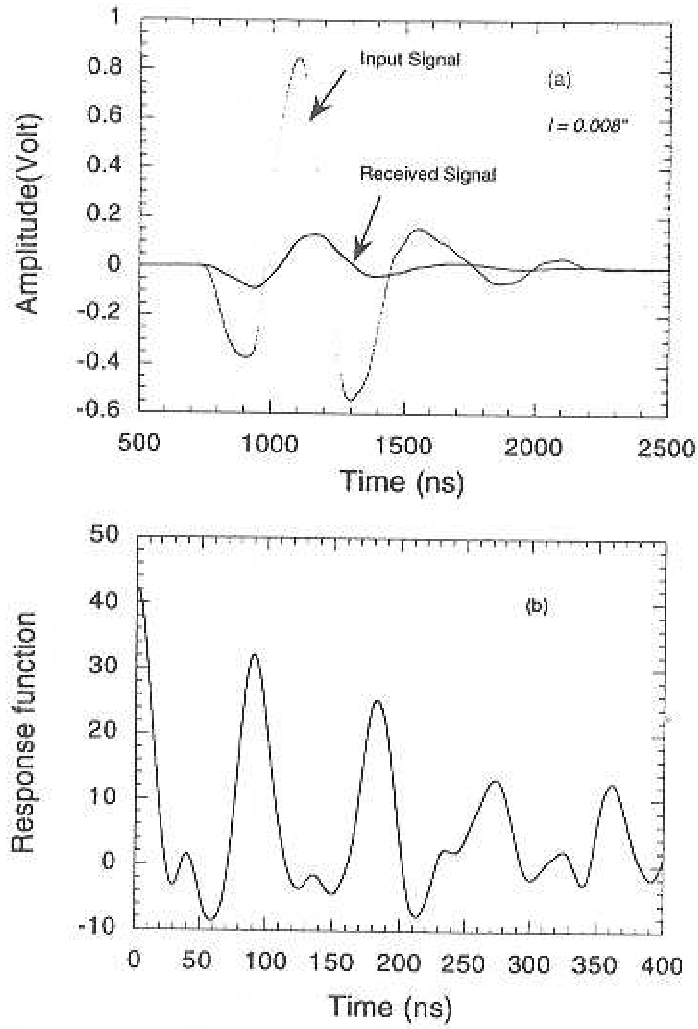

FIGURE 4 (a) The input signal to the sample and the received signal after the pulse went through the sample and (b) The response function derived from (a). The sample thickness is 0.008″.

observed independently. The time shift between the input and the transmitted signals cannot be measured directly because the time difference is too small. Figures 3(b) and 4(b) are the calculated corresponding response functions for the two cases. The separation between the peaks of the response function is the time lagging between consecutive reflections inside the sample.

The sound velocity calculated using time difference between the peaks of the response function is given in Table I. One can see that the measurements are very consistent and accurate. The average sound velocity is 4480.4 m/s which gives a value for the elastic constant $c_{11} = 17.0\times10^{10}$ N/m^2. No obvious change of the elastic property with size was observed in this thickness range as shown in Table I.

TABLE I Longitudinal velocity and elastic constant c_{11} for brass shims. (70% Brass, 30% Zinc; $\rho = 8.4697$ g/cm^3)

Sample Thickness (inch)	*Longitudinal Velocity (m/s)*	$c_{11}(10^{10} N/M^2)$
0.004″	4431.8	16.635
0.005″	4519.6	17.301
0.006″	4456.1	16.818
0.008″	4490.6	17.080
0.015″	4526.7	17.355
0.025″	4457.7	16.830

During measurements, the number of observations used for noise spectrum calculation are: $m_1 = 5$ and $m_2 = 10$. Minor changes were found when the averages were taken at different m_1 and m_2 number of observations but is well within the experimental error. Using the center frequency of the transducer as a measure, the 0.004″ thick sample is less than 5% of the wavelength $\lambda = 2.24$ mm.

As shown in Figures 3 and 4 the response function is far from a δ-function series, however, one can clearly distinguish the peak positions. It is found that the response function becomes less sensitive as the thickness decreases. The peaks of the response function are sharper in the case of the 0.008″ thick sample than in that of the 0.004″ thick sample as shown in Figures 3 and 4. Using the 2 MHz transducer, the method reaches its limit at 0.003″. All the peaks got smeared out when the sample becomes thinner than 0.002″.

SUMMARY AND CONCLUSIONS

A data processing scheme is introduced here to extend the resolution of the ultrasonic pulse-echo method. Using the filtered response function and a 2 MHz broadband transducer, the sound velocity was measured for a set of brass samples and the thinnest sample is less than 5% λ of the center frequency. This result is very encouraging since the sub-wavelength resolution is the only way to resolve the intrinsic conflict of high resolution and deep penetration in ultrasonic technology. This technique may be used to study the bonding layer in composite structures and interfaces. Further extension of the method to higher frequencies and multi-layer structure is under investigation.

Acknowledgment

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