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NONUNIFORM SURFACE DEFORMATION OF PIEZOELECTRIC COMPOSITES AND THE EFFECT OF SURFACE PLATES

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ABSTRACT

A theoretical model is presented for the piezoelectric ceramic-polymer composites with 2-2 connectivity. Based on a simple assumption, analytical solutions can be obtained for the inhomogeneous surface displacement profile. The theory is extended to address the case of adding surface plates to the composite. Variations of the surface profiles with the change of ceramic aspect ratio, spacing of ceramic, Young's modulus of the polymer and the thickness of surface plates are also calculated.

INTRODUCTION

Piezoelectric ceramic-polymer composites have shown great advantages as transducer materials in the area of ultrasonic imaging and underwater acoustics (Smith, 1991, Oakley, 1991). Through interfacial stress transfer between the two constituents, the composite structure efficiently utilized the piezoelectric property of the hard ferroelectric ceramic and the low density of the soft polymer, resulting in a product which is superior to each of the individual components. Experiments showed that the effective piezoelectric properties can be as high as 80% of the ceramic value even for as little as 30% of ceramic volume content. In addition, the ability to adjust the effective mechanical and the electric properties of the material solely through varying the ratio and the geometries of the constituents make the piezoelectric composite a very attractive candidate for many sensing and actuating applications.

Many studies have been carried out to study the functional mechanism in the composite and attempt to optimize the relevant physical properties for particular applications. However, due to the complexity of the elasto-electric differential equations, one often has difficulties to solve rigorously the complete coupled elasticity equations. The existing theoretical models may be categorized into three different types: 1. approximate solutions(Skinner et al, 1978, Smith et al. 1985, Chan and Unsworth, 1989, Jensen, 1991); 2. effective medium theory(Auld, 1984, Auld and Wang, 1984); and 3. finite element analysis(Hossack and Hayward, 1991).

Although there are advantages and disadvantages in any given method, the approximate solutions give the most direct and clear physical insight. Some progresses along this line will be discussed in this paper and some new results will be presented.

THE MODEL

The simplest approximation can be made for the piezocomposite is to assume a uniform deformation of both constituents[Fig. 1(a)]. Although this assumption is obviously an over simplification, many features can be described qualitatively, and it gives the right trend for the variation of the physical properties of the composites with the change of the ceramic content. But, the magnitude of the calculated values are often larger than the experimental values. This is due to the fact that the two constituents actually deform nonuniformly under either electric field or stress. A typical surface profile is schematically shown in Fig. (1b). The nonuniformity becomes very severe when the ceramic content becomes very low. The non-piezoelectric polymer phase has nearly zero deformation sufficiently far away from the ceramic. This fact has been long recognized (Klicker, 1980) and some numerical works using finite element analysis also gave similar results. However, the underlying physics, especially the stress transfer mechanism was not conceptually very clear.

Recently we have developed a model treating this nonuniform deformation (Cao, Zhang and Cross, 1992, 1993),

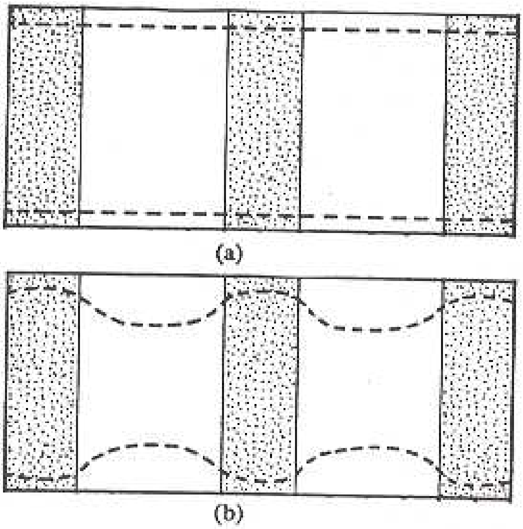

Fig. 1. Schematic plot of the surface displacement profile in a 2-2 composite. (a) is the isostrain approximation. (b) is the actual nonuniform deformation.

which gives good quantitative prediction on the physical properties of the composite structure. The model is briefly introduced below. A structural element is chosen as shown in Fig. 2 for a 2-2 composite, where a and d are the dimensions of the piezo-ceramic and the polymer respectively in the x-direction, and l is the thickness of the composite in the z-direction. If the y-dimension, h, is very large, the system will be effectively confined in that dimension. When the piezo-ceramic is driven electrically, the polymer will be pulled up

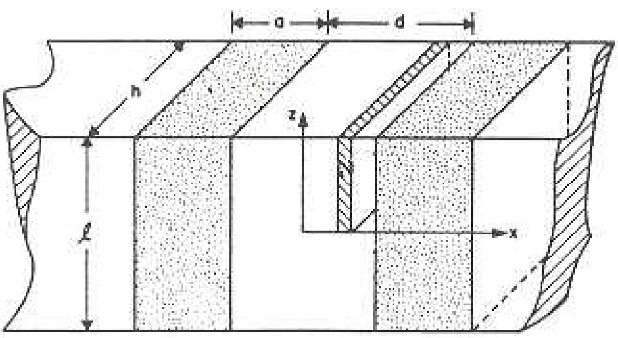

Fig. 2. A section of the 2-2 composite. The dimensions and the coordinate system are also given in the plot.

and down by the ceramic through a shear stress at the ceramic polymer interface, the magnitude of this shear stress will vary in the x-direction and is proportional to the displacement difference between the two constituents.

Since we are mainly interested in the nonuniform surface deformation, for a linear system, it is reasonable to assume that the strain is uniform in the z-direction for any given x (Cao, Zhang and Cross, 1992). While this approximation as an integration average may not be exact for the interior of the system, it is reasonably good for the study of surface displacement. With this approximation we can convert this problem into quasi-one-dimensional and write down the equilibrium condition at the surface of the composite(Cao, Zhang and Cross, 1992),

$$\frac{l}{4}\mu^P u_{xx}(x,l/2) - \frac{2}{l}Y^P u(x,l/2) = 0, \quad -\frac{d}{2} < x < \frac{d}{2}; \qquad (1)$$

$$\frac{l}{4}c_{44} u_{xx}(x,l/2) - \frac{2}{l}\frac{1}{s_{33}} u(x,l/2) + e_{33}E = 0, \quad \frac{d}{2} < x < \frac{d}{2} + a. \qquad (2)$$

where μ^P and Y^P are the shear modulus and Young's modulus of the polymer, c_{44} and s_{33} are the shear elastic constant and the normal compliance of the ceramic, respectively, e_{33} is the piezoelectric stress constant of the ceramic.

Eqs.(1) and (2) can be solved analytically,

$$u(x, l/2) = A\, \mathrm{ch}\left(\frac{2}{l}\sqrt{\frac{2Y^P}{\mu^P}}\; x\right), \quad -d/2 < x < d/2; \qquad (3)$$

$$u(x, l/2) = B\, \mathrm{ch}\left[\frac{2}{l}\sqrt{\frac{2}{s_{33}c_{44}}}\left(x - \frac{a+d}{2}\right)\right] + \frac{l}{2}s_{33}e_{33}E, \quad d/2 < x < d/2 + a; \qquad (4)$$

$$u(x + n[a+d], l/2) = u(x, l/2), \quad n=1, 2, 3 .. \qquad (5)$$

Eq. (5) represents the periodic condition of the system. The coefficients A and B can be determined from the nonslip interface boundary condition and the force balance condition (Cao, Zhang and Cross, 1992).

$$A = \frac{\frac{l}{2}s_{33}\, e_{33}E}{\sqrt{\frac{\mu^P Y^P s_{33}}{c_{44}}}\ \mathrm{sh}\left(\frac{d}{l}\sqrt{\frac{2Y^P}{\mu^P}}\right)\mathrm{cth}\left(\frac{a}{l}\sqrt{\frac{2}{s_{33}c_{44}}}\right) + \mathrm{ch}\left(\frac{d}{l}\sqrt{\frac{2Y^P}{\mu^P}}\right)} \qquad (6)$$

$$B = \frac{-\frac{l}{2} s_{33}\, e_{33}\, E}{\sqrt{\frac{c_{44}}{\mu^P Y^P s_{33}}}\ sh\left(\frac{a}{l}\sqrt{\frac{2}{s_{33}\, c_{44}}}\right) cth\left(\frac{d}{l}\sqrt{\frac{2\, Y^P}{\mu^P}}\right) + ch\left(\frac{a}{l}\sqrt{\frac{2}{s_{33}\, c_{44}}}\right)} \qquad (7)$$

Based on the inhomogeneous solutions of the surface deformation, Eqs. (3) and (4), we can derive the physical properties of the composite from the properties of the two constituents. As shown by Cao, Zhang and Cross (1992) that the physical properties of the composite depend strongly on the aspect ratio of the ceramic and the polymer. For fixed a and d, larger l gives better stress transfer. The surface nonuniformity also reduces as the thickness of the composite, l, increases. The upper limit for the stress transfer can be achieved only at the isostrain condition, i.e., the ceramic and polymer deform uniformly. Also, the polymer phase should have smaller Young's modulus but large shear modulus for better stress transfer from the polymer to the ceramic and reduce the self loading of the polymer phase.

This fact has been recognized from empirical testings. In practice, surface plates are often used at the top and bottom surfaces to improve the surface uniformity and to increase stress transfer between the two constituents, surface plates effectively increase the shear modulus of the system. However, surface plates introduces additional interface which complicates the structure, and additional mass which not only reduces the effective volume of the structure but also changes the acoustic impedance of the system. Therefore, it is important to know the effect of the surface plates and how do they change the surface deformation.

SURFACE PLATES

Surface plates enforce deformation uniformity in the two constituents by adding additional stress transfer through the plates. The degree of uniformity depends on the thickness of the plate and the ratio of the Young's modulus between the polymer and the plate. For simplicity, we treat the surface plate as a thin plate problem. From the standard analyses(Landau and Lifshitz, 1959) we can write down the new equilibrium condition at the surface of the composite:

$$D\, u_{xxxx} + \frac{l}{4}\mu^P u_{xx}(x, l/2) - \frac{2}{l} Y^P u(x, l/2) = 0, \quad -\frac{d}{2} < x < \frac{d}{2}; \qquad (8)$$

$$D\, u_{xxxx} + \frac{l}{4} c_{44}\, u_{xx}(x, l/2) - \frac{2}{l}\frac{l}{s_{33}} u(x, l/2) + e_{33} E = 0, \quad \frac{d}{2} < x < \frac{d}{2} + a. \qquad (9)$$

where

$$D = \frac{Y^S}{12\,(1-\sigma^2)t^3} \qquad (10)$$

Y^S and σ are the Young's modulus and the Poisson's ratio of the surface plate, t is the thickness of the surface plate.

Formally eqs.(8) and (9) still have the same solutions as Eqs.(3)-(5)

$$u(x, l/2) = A\, ch(\beta^P x), \quad -d/2 < x < d/2; \qquad (11)$$

$$u(x, l/2) = B\, ch\left[\beta^C \left(x - \frac{a+d}{2}\right)\right] + \frac{l}{2} s_{33} e_{33} E, \quad d/2 < x < d/2 + a; \qquad (12)$$

where

$$\beta^P = \sqrt{\frac{-l\,\mu^P + \sqrt{l^2 \mu^{P2} + 128\, D\, Y^P/l}}{8D}} \qquad (13)$$

$$\beta^C = \sqrt{\frac{-l\, c_{44} + \sqrt{l^2 c_{44}^2 + 128\, D/s_{33}\, l}}{8D}} \qquad (14)$$

The two integration constants A and B can be determined from the continuity requirement for u(x, $l/2$) at x=d/2 and the force balance condition at the surface,

$$A = \frac{\frac{l}{2} s_{33}\, e_{33}\, E}{\frac{\beta^c}{\beta^P} Y^P s_{33}\, sh(\beta^P \frac{d}{2})\, cth(\beta^c \frac{a}{2}) + ch(\beta^P \frac{d}{2})} \qquad (15)$$

$$B = \frac{-\frac{l}{2} s_{33}\, e_{33}\, E}{\frac{\beta^P}{\beta^c\, Y^P s_{33}}\, sh(\beta^c \frac{a}{2})\, cth(\beta^P \frac{d}{2}) + ch(\beta^c \frac{a}{2})} \qquad (16)$$

RESULTS AND DISCUSSIONS

From the solutions derived above, we can plot the variation of the nonuniform surface displacement profile with respect to the aspect ratio and the thickness of the surface plate. Fig. 3 shows the surface nonuniformity of a PZT5A-Spurrs Epoxy composite system under an electric potential of 0.5 V. Material properties used in this and the following calculations are listed in Table I

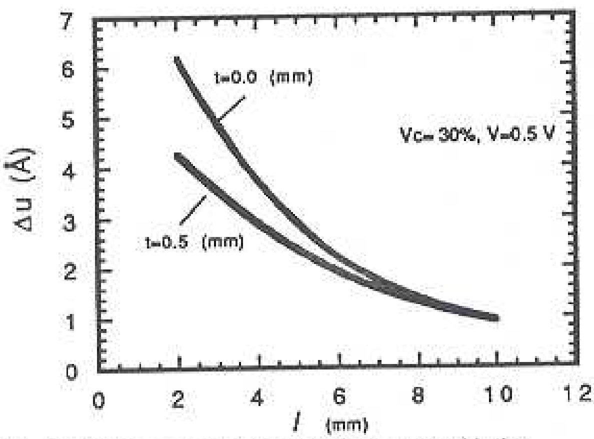

Fig. 3 Surface nonuniformity decreases with the increase of aspect ratio. a is chosen to be 1 mm. The lower curve is for a composite with 0.5 mm thick surface plate. Vc is the ceramic volume content.

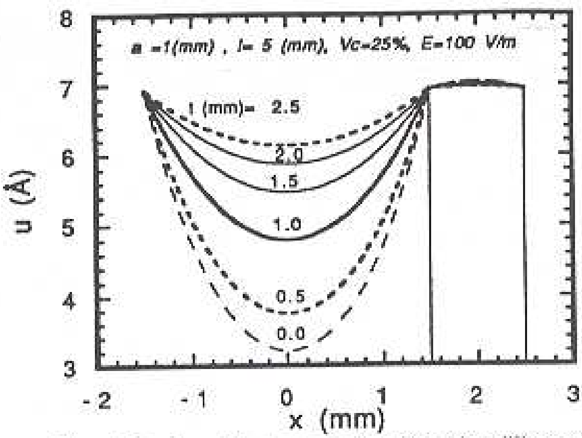

Fig. 4. Surface displacement profiles for different surface plate thickness in one period of a 2-2 composite.

Δu is the maximum surface displacement difference which is defined by the following equation:

$$\Delta u = \frac{1}{2} s_{33}\ e_{33}\, E + B - A. \qquad (17)$$

Since the system is linear for low field, the displacement will be independent of the thickness of the composite. As we can see from Fig. 3 that the surface uniformity increases with the increase of the aspect ratio of the ceramic for a given ceramic content (30% by volume). Fig. 4 shows the improvement of the surface uniformity with the increase of the thickness of the surface plate for a ceramic aspect ratio of 5:1 and a ceramic volume content of 25%. Due to the additional stress transfer through the surface plates, the surface displacement becomes more uniform. The improvement is drastic at the beginning, and gradually saturated as the surface deformation approaching uniform.

Fig. 5 is the variation of Δu versus surface plate thickness calculated for two different ceramic volume content (25% and 50%). We can see that the change becomes very slow in both cases for large t. For low ceramic content, surface uniformity is difficult to achieve.

For a fixed ceramic aspect ratio, the ceramic volume content specifies the distance between the ceramic plates. Surface uniformity can be drastically improved by placing the ceramic plates sufficiently close. This is also shown in Fig. 5. The 50% ceramic content sample has much smaller Δu.

From Eqs. (15)-(17) it is easy to see that the uniformity, aspect ratio, the Young's modulus of the inactive phase, and the surface plate thickness are all interelated. Because the Young's modulus of the polymer determines the level of loading on the surface plate, it has very strong influence on the surface uniformity. Fig. 6 shows the variation of the surface displacement profile with respect to the change of

TABLE I. Material properties used in the this paper

PZT:

c_{33}=11.086 (10^{10} N/m^2), c_{44}=2.07 (10^{10} N/m^2)

c_{13}= 7.5 (10^{10} N/m^2), s_{33} =0.18 (10^{-10} m^2/N)

d_{33}= 374 (10^{-12} C/N), d_{31}= -171 (10^{-12} C/N)

Spurrs Epoxy:

Y^p = 4.796 (10^9 N/m^2), μ^p = 1.758 (10^9 N/m^2)

Brass plate:

Y^s=9.0 (10^{10} N/m^2), σ=0.31

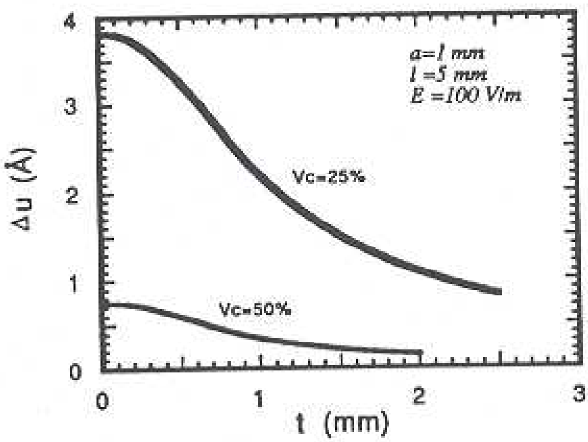

Fig. 5. The maximum non-uniform deformation for different surface plate thickness at two different ceramic volume content.

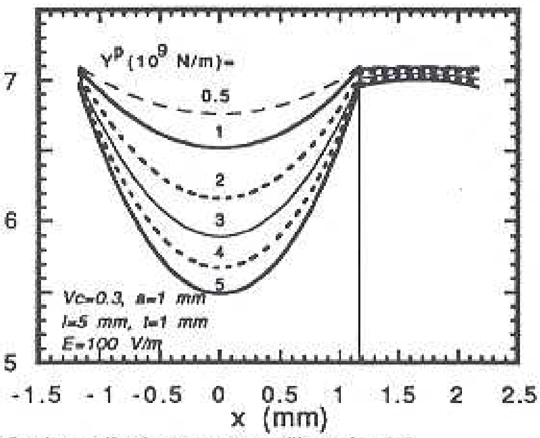

Fig. 6. Surface displacement profiles of a 2-2 composite in one period with different polymer Young's modulus.

Young's modulus of the polymer phase for a surface plate thickness of 1 mm. We can see that the improvement for uniformity is quite drastic when the Young's modulus of the polymer is reduced.

Since lowering the Young's modulus can be easily achieved by blend air bubbles into the polymer matrix, it offers a very useful way to achieve surface uniformity. One may also notice in Fig. 6 that the average effective displacement of the composite increases as the Young's modulus of the polymer is reduced due to the reduction of the loading produced by the polymer.

CONCLUSIONS

A theoretical model is presented to calculate the nonuniform surface displacement profile in a 2-2 piezoelectric ceramic-polymer composite. Because the inactive nature of the polymer phase, the surface displacement can be quite nonuniform. This non-uniformity can be improved through several different methods: 1. put surface plates, 2. reduce the effective Young's modulus of the polymer, 3. increase the aspect ratio of the ceramic, and 4. increase the ceramic content. In practice, one may need to utilize all of the four ways to balance the need for structural stability, low acoustic impedance and surface uniformity.

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