Effects of Face Plates on Surface Displacement Profile in 2-2 Piezoelectric Composites

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***Abstract*—A simple model is developed to describe the inhomogeneous surface deformation profile of face plated 2-2 type piezocomposites. The contribution of face plate to the equilibrium condition is approximated as from simple elastic bending of the plate. Analytical solutions were obtained for the inhomogenous surface displacement profile. From these solutions one can predict the variation of the nonuniform surface displacement in a 2-2 composite with respect to material and geometry parameters. It is shown that the surface displacement uniformity depends on several factors: the ceramic aspect ratio, the spacing between ceramic plates, the thickness of face plate, the Young's modulus of the polymer and of the face plate. The calculated results indicate that stiffer face plates, softer polymer resin, and closer ceramic spacing could make the piezocomposite transducers to have more uniform surface displacement.**

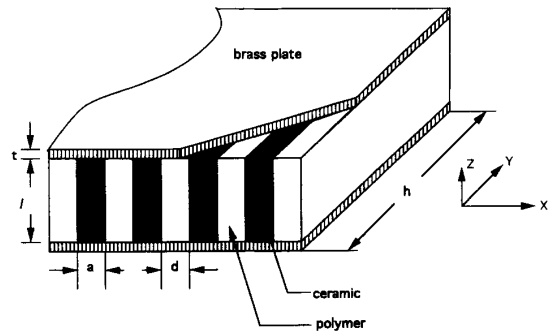

Fig. 1. Schematic of a face plated 2-2 piezocomposite and the coordinate system used in the calculations.

I. INTRODUCTION

ONE of the key features of piezoelectric composites is the stress transfer capability between the hard ceramic and the soft polymer, which gives the composite a high level of piezoelectric capability and at the same time, lowers the effective acoustic impedance of the composite to make it more suitable for underwater and medical applications [1]–[4]. The polymer phase can also reduce the Q-value of the transducer to suppress ringing. However, as reported in our early works [5]–[7], the difference in elastic stiffness between the two constituents in the composite causes surface displacement to be nonuniform under external (electric or elastic) fields. This nonuniformity reduces the efficiency of stress transfer between the two constituents, hence degrading the piezoelectric performance of the transducers, and causing the physical properties of piezocomposites to depend on the aspect ratio of the ceramic and their spacing. In some actuator applications, such as short wavelength plane wave generators, uniform surface displacement is preferred. According to our previous analyses [5], more uniform surface displacement requires the polymer to have large shear modulus but small Young's modulus, which is difficult to achieve since the Poisson's ratio for most of the known materials is between 0.3–0.4. A common practice to overcome this problem is to add stiff face plates to the composites. The question is how to determine the proper thickness of the face plates and how to choose the face plate material, considering the fact that face plates are not piezoelectric, which effectively increases the inactive volume of the composite. In addition, other properties, such as the acoustic impedance, mechanical loss and structural stability will all be affected by the addition of face plates. It is therefore important to understand and evaluate the face plate effects and try to optimize the selection of face plate material and geometry for practical applications. In this paper we extend the model of [5] by including the contribution of the face plates in constructing the equilibrium condition at the composite surface. Brass was used as the face plate material in our calculations, but the procedure can be easily generalized for face plates made of other materials. A comparison with stainless steel and GRP (glass reinforced polymer) face plates is briefly made.

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II. THE MODEL

The 2-2 lamellar composite and the coordinate system used in the calculations are given in Fig. 1, where z is the poling direction, a and d are the width of the ceramic and polymer, respectively. We choose the system with $h \gg l$, so that the properties of the system may be considered to be independent of y. The origin of the x-coordinate is set at the center of one of the polymer plate. In a previous paper [5], a linear model for the 2-2 composite without face plates was reported. In that model we have assumed the strain component along the z-direction to be constant for any given x. Our recent analysis using series expansion for the displacement field [8] and results from finite element analyses show that the strain

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is quite uniform along z-direction except near the surface region.

The addition of face plate not only reduces the surface displacement nonuniformity, but also forces the strain to be more uniform inside the composite, making the constant z-strain a better assumption. Based on this consideration, as a first attempt to model the face plated composite, we extend our previous work [5]. The equilibrium condition at the surface of a 2-2 composite was given there; all we need to do here is to add the contribution of the face plates. This is accomplished by treating the face plate deformation as simple elastic bending of a thin plate. The force density generated from the face plat is Du_{xxxx} [7], where u is the surface displacement in the z-direction, the subscript x represents derivative with respect to the coordinate x, and D is the flexural rigidity of the face plate,

$$D = \frac{Y^f t^3}{12(1-\sigma^2)} \tag{1}$$

In (1) Y^f and σ are, respectively, the Young's modulus and the Poisson's ratio of the face plate, and t is the thickness of the face plate.

The equilibrium condition at the composite surface $z = l/2$ is simply the addition of the Du_{xxxx} term to the equilibrium condition (7) of [5],

$$Du_{xxxx}(x,l/2) + \frac{l}{4}\mu^p u_{xx}(x,l/2) - \frac{2}{l}Y^p u(x,l/2) = 0, \quad -\frac{d}{2} < x < \frac{d}{2} \tag{2}$$

$$Du_{xxxx}(x,l/2) + \frac{l}{4}\mu^c u_{xx}(x,l/2) - \frac{2}{l}Y_{33}^c u(x,l/2) + Y_{33}^c d_{33}E = 0, \quad \frac{d}{2} < x < \frac{d}{2} + a \tag{3}$$

where μ^p and Y^p are the shear modulus and Young's modulus of the polymer, $\mu^c\,(= c_{44})$ and $Y_{33}^c\,(= 1/s_{33})$ are the shear modulus and Young's modulus of the ceramic, respectively. d_{33} is the piezoelectric constant of the ceramic, E is the electric field along the poling direction, i.e., z-direction.

Equations (2) and (3) are the static equilibrium condition at the surface of the composite, which can be solved analytically to give the hyperbolic cosine solutions:

$$u(x,l/2) = A\cosh(\beta^p x), \quad -d/2 < x < d/2 \tag{4}$$

$$u(x,l/2) = B\cosh\left[\beta^c\left(x - \frac{a+d}{2}\right)\right] + \frac{l}{2}d_{33}E, \quad \frac{d}{2} < x < \frac{d}{2} + a \tag{5}$$

$$u(x + n[a+d], l/2) = u(x, l/2), \quad n = 1,2,3\cdots \tag{6}$$

where

$$\beta^p = \sqrt{\frac{-l\mu^p + \sqrt{l^2\mu^{p^2} + 128DY^p/l}}{8D}} \tag{7}$$

$$\beta^c = \sqrt{\frac{-l\mu^c + \sqrt{l^2\mu^{c^2} + 128DY_{33}^c/l}}{8D}} \tag{8}$$

Equation (6) represents the periodic boundary condition of the system. Here we have selected the solution so that it recovers the solution of [5] in the limit of $t \to 0$. The coefficients A and B can be determined from the nonslip interface boundary condition and the force balance condition at $x = d/2$ [5]:

$$A = \frac{\frac{l}{2}d_{33}E}{\frac{\beta^c}{\beta^p}\frac{Y^p}{Y_{33}^c}\sinh\left(\beta^p\frac{d}{2}\right)\coth\left(\beta^c\frac{a}{2}\right) + \cosh\left(\beta^p\frac{d}{2}\right)} \tag{9}$$

$$B = \frac{-\frac{l}{2}d_{33}E}{\frac{\beta^p Y_{33}^c}{\beta^c Y^p}\sinh\left(\beta^c\frac{a}{2}\right)\coth\left(\beta^p\frac{d}{2}\right) + \cosh\left(\beta^c\frac{a}{2}\right)} \tag{10}$$

Based on the inhomogeneous solutions (4) and (5) for the surface displacement we can derive the physical properties of the composite from the properties of the three constituents, i.e., ceramic, polymer and face plates. As shown in our earlier works [5]–[7], without face plates, the physical properties of composites depend strongly on the aspect ratio of the ceramic and their spacing. This aspect ratio dependence is a direct consequence of the displacement inhomogeneity in the polymer and the ceramic, which is in turn produced by the active and passive nature of the two constituents. It is expected that the addition of face plates to a composite will enforce deformation uniformity. In contrast to the stress transfer in a nonface plated composite for which the transferred stress is pure shear stress, the additional stress transferred between the ceramic and the polymer via the face plate is primarily a normal stress in the z-direction. The degree of uniformity in a face-plated composite depends on several factors: the thickness of the plate, the Young's modulus of the polymer, the Young's modulus of the face plate, the ceramic content as well as the ceramic aspect ratio and element spacing. All of these factors are now included in the solutions (4) and (5), which makes it very convenient to evaluate the influence of each material parameter. As an example, we have calculated the inhomogeneous surface displacement profiles for a PZT5H-Spurrs epoxy composite with brass face plates under an electric field E. Parameters were varied to show the general trend for the optimization of the composite configuration. The material constants used in the calculations are given in Table I.

Fig. 2 shows the calculated inhomogeneous surface displacement variations as a function of the increase of face plate thickness t. The dimensions of the composite used in the calculations are: $l = 5$ mm, $a = 1$ mm, and $d = 2$ mm. A voltage of $\sqrt{2}$ volts is applied to the sample along the z

TABLE I
ELASTIC, PIEZOELECTRIC AND DIELECTRIC P ROPERTIES OF PZT 5H, B RASS PLATE AND S PURRS EPOXY USED IN O UR CALCULATIONS .

	Y 10^{10} N/m^2	μ 10^{10} N/m^2	d_{33} 10^{-12} C/m	d_{31} 10^{-12} C/m	σ
PZT5H	11.74	2.3	593	-274	.364
SpurrsEpoxy	.48	.18			
Brass	9.0				.31

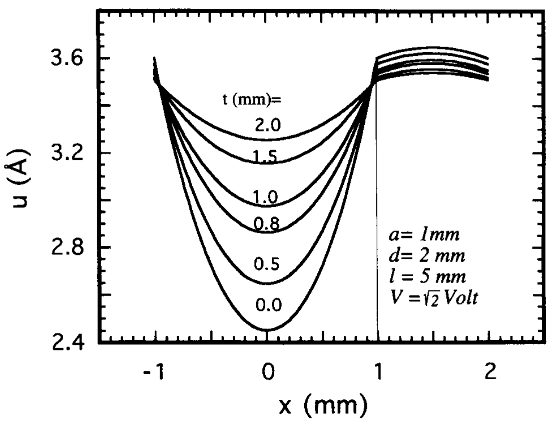

Fig. 2. Calculated surface displacement profiles for a PZT5H-Spurrs epoxy 2-2 composite with different face plate thickness. The volume fraction of the ceramic is 1/3.

-direction (poling direction). For $t = 0$ (i.e., no face plates), the displacement at the center of the polymer surface differs substantially from the displacement at the center of the ceramic surface. When $t > 0$, the polymer surface displacement increases rapidly with the increase of the face plate thickness, and at the same time, the displacement of the ceramic is somewhat reduced. Because the effect of the face plate is to make the polymer move more and the ceramic move less, the overall composite surface displacement becomes more uniform as shown in Fig. 2. One notices that the surface displacement changes caused by the addition of face plates appear mainly in the polymer, which is due to the large difference in elastic stiffness between the polymer and the ceramic. The improvement on the surface displacement uniformity becomes less as the isostrain condition is approached. We believe for a brass face plate thickness greater than 2 mm in this configuration, the physical properties can be well accounted for by the isostrain approximation [4].

Without a face plates, composites made of soft polymer will have more severe displacement nonuniformity than composites made of hard polymer because soft polymers have a smaller shear modulus which cannot effectively transfer stress between the polymer and the ceramic. After adding face plates to the composite, the situation is reversed. Composites made of softer polymer will have more uniform displacement than composites made of harder polymer. This is due to the fact that the additional stress transferred by the face plate from the ceramic to the polymer is in the form of a normal stress. Both the ceramic and the polymer interact directly with the face plates.

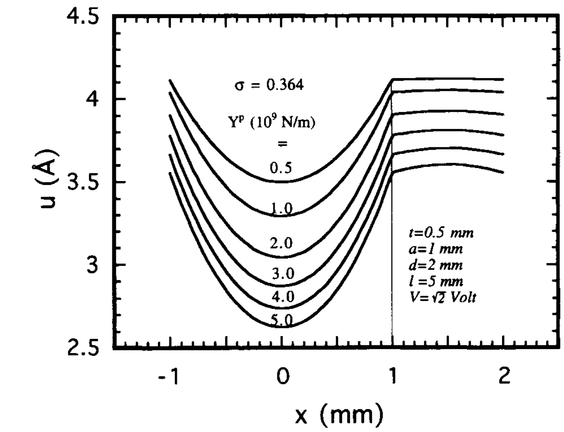

Fig. 3. Calculated surface displacement profiles for different Young's modulus of the polymer. The Poisson's ratio for the polymer is kept at $\sigma^P = 0.364$. Ceramic volume fraction is 1/3 and the face plate thickness is $t = 0.5$ mm.

Since the polymer phase is nonpiezoelectric, it adds to the loading on the ceramic phase, the level of this loading is proportional to the stiffness of the polymer. In other words, a softer polymer has less resistance to elastic deformation, therefore, will be easier to be driven toward more uniform displacement with the ceramic phase with the help of face plates. This situation is illustrated in Fig. 3 where the surface displacement is plotted for different elastic stiffnesses of the polymer. The face plate thickness was kept constant for these calculations at $t = 0.5$ mm and the Poisson's ratio for the polymer is fixed at 0.364. Since the elastic stiffness of different types of polymers can easily differ by one order of magnitude, it is relatively easy to control this parameter. One can see from Fig. 3 that the surface displacement uniformity is improved substantially by reducing the Young's modulus of the polymer, and more importantly, the total effective displacement of the composite is also increased due to the reduction of the self loading produced by the polymer. For air kerf (infinitely soft resin) face plated composite the displacement would be uniform. When the polymer is stiff, the ceramic surface shows noticeable curvature, but for very soft polymer composites, only the polymer phase shows nonuniform surface displacement while the ceramic surface is practically flat as shown in Fig. 3.

Another important issue is the selection of the face plate material. From (4) and (5) one can draw the conclusion that stiffer materials are preferred for the purpose of achieving more uniform surface displacement. For comparison, we have calculated the surface displacement profile for three different face plate materials: steel, brass and GRP, and the results are shown in Fig. 4. We found that stiffer face plate does improve the uniformity of the surface displacement, however, the effect of using a stiffer face plate is much less than reducing the Young's modulus of the polymer.

Under the application of an electric field E, the maximum surface displacement difference, Δu, between the center of the polymer and the center of the ceramic can be derived using (4) and (5),

$$\Delta u = \frac{l}{2} d_{33} E + B - A. \qquad (11)$$

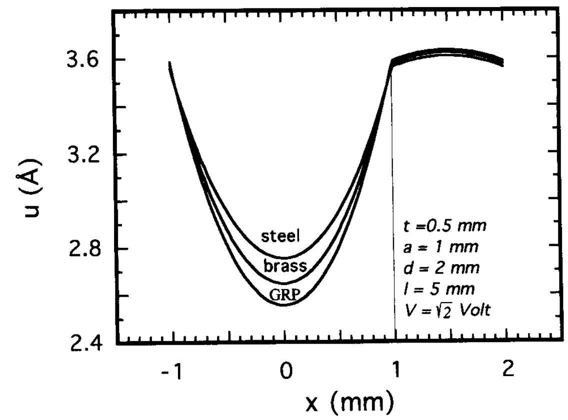

Fig. 4. The surface displacement variation caused by the change of face plate material. The face plate thickness is chosen as $t = 0.5$ mm.

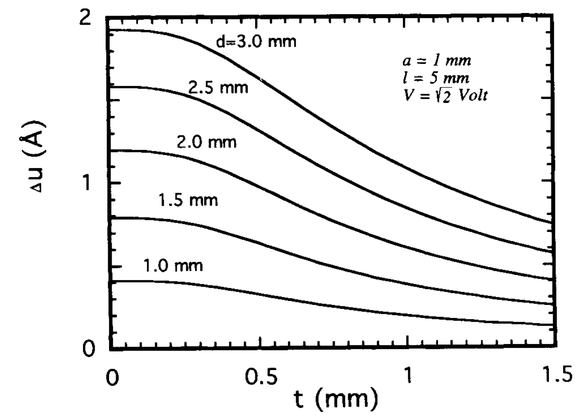

Fig. 5. The calculated maximum displacement difference Δu for different face plate thickness t in a 2-2 composite with ceramic spacing d as a parameter. The ceramic plate thickness is $a = 1$ mm.

This quantity can be used as a measure of the uniformity in surface displacement. Fig. 5 shows Δu versus face plate thickness for a 2-2 composite calculated at five different volume contents. The ceramic plate thickness is fixed at 1 mm in the calculations. The results show that the improvement on the surface displacement uniformity becomes less effective after the brass face plate thickness is beyond certain limit. The effects of face plate is more pronounced for composites made of large spaced ceramics, or large d-value. One of the important conclusions should be mentioned is that the ceramic spacing plays more important role than the face plates in terms of making the surface displacement more uniform. This is clearly seen in Fig. 5 for composites without face plates ($t = 0$).

III. Summary and Conclusions

A theoretical model is proposed to calculate the surface displacement profile in face plated 2-2 piezoelectric ceramic-polymer composites. Predictions on the influence of geometry and material properties of each constituents to the surface displacement uniformity are given. It is concluded that the nonuniform displacement in face plated composites can be improved by several methods: (a) increase face plate thickness; (b) increase the Young's modulus of the face plate; (c) reduce the Young's modulus of the polymer; (d) increase the ratio of l/a; and (e) increase the ceramic volume ratio. Both (d) and (e) can reduce the spacing between the ceramics.

The addition of face plates to the composite structure makes it possible to use softer polymer resin, which can reduce the polymer loading and improve the effective electromechanical conversion property of the composite. Face plates allows normal stress transfer along z-axis between the ceramic and the polymer, which makes the overall surface displacement of the composite more uniform and the interior deformation more close to isostrain condition.

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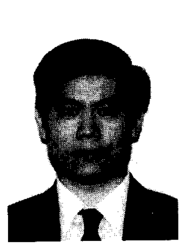

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