Homework 7, Due October 17th

1-3. Problems 1-3 from Evans. pp.162-163.

4. (a) Show that the initial value problem for $u(t,x)$ on $\mathbb{R}_+ \times \mathbb{R}$,

$$\begin{aligned} u_t + \left(\frac{u^3}{3}\right)_x &= 0 \qquad \text{for } t > 0 & (1)\\ u(0,x) &= x\,, & (2) \end{aligned}$$

has no local (smooth) solution.

(b) Solve the same equation with data

$$u(0,x) = \begin{cases} -1 & \text{for } x \le -1 \\ x & \text{for } -1 \le x \le 1 \\ 1 & \text{for } x \ge 1\,, \end{cases}$$

Hint for part a) observe that (global) solutions do not exist for some $t_0 > 0$, if you can find two (distinct) characteristics that cross at t_0 (or earlier).

5. In the x-t plane, find all C^2 solutions of $xu_t - tu_x = 0$ with initial conditions $u(x,1) = x^2$.

Note: For exactly the same problem in class I gave you an example of a (second) solution which was continuous, but it was not C^2. Thus it could not be classical. This problem asks you to construct classical solutions.