Homework 2, Due September 12th

1. Problem 2 (Chapter 2) on p.85 of Evans's book.
2. Problem 3 (Chapter 2) on p.86 of Evans's book.
3. Problem 4 (Chapter 2) on p.86 of Evans's book.
4. Assume $u \in C^2(\bar{\Omega})$ solves

$$\Delta u = f(u)u, \ u|_{\partial\Omega} = 0$$

for some non-negative continuous function f such that $f(u) > 0$ for any $u \neq 0$ (for example, $f(u) = u^2$). Prove that u must be identically zero: $u \equiv 0$.
Hint: Prove that u must be subharmonic/superharmonic (see Problem 3 in this assignment) near its (potential) interior minimum/maximum, respectively.

5. Suppose u_1 and u_2 are solutions of a Poisson equation with different right-hand side:

$$-\Delta u_1 = f_1(x), \ u_1|_{\partial\Omega} = 0, \ -\Delta u_2 = f_2(x), \ u_2|_{\partial\Omega} = 0.$$

Prove that if $f_1(x) \geq f_2(x)$ for every $x \in \Omega$, then $u_1(x) \geq u_2(x)$.
Note: This property is called *positivity*: the inverse of the Laplacian is a positive operator.